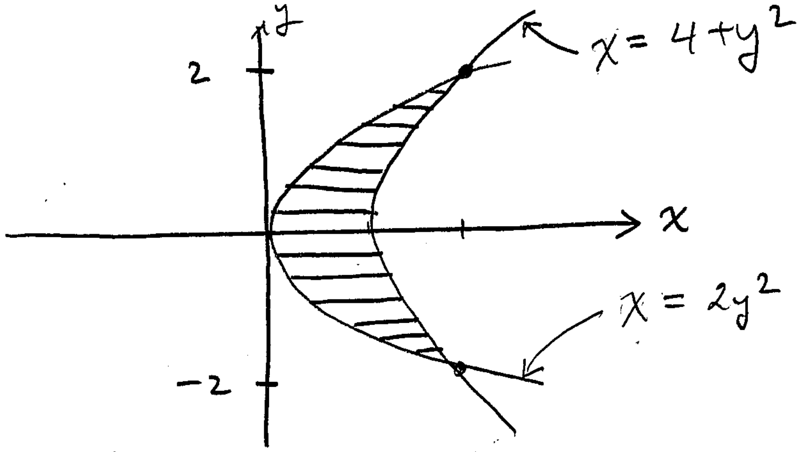
Math 122 F09N01 - Quiz 6 Name: SOLUTIONS.
Nov 20 2009 Stu#:

(1) **[15 points]** Determine the area bounded by the curves $x = 2y^2$ and $x = 4 + y^2$. First sketch the two given curves and determine the points of intersection.



Points of Intersection:

$$2y^2 = 4 + y^2$$
$$y^2 = 4$$
$$y = 2, -2$$

$$\therefore \quad A = \int_{y=-2}^{2} (4+y^2) - (2y^2)\, dy$$

$$= \int_{-2}^{2} 4 - y^2\, dy$$

$$= \left[4y - \frac{y^3}{3}\right]_{-2}^{2}$$

$$= \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)$$

$$= \boxed{\frac{32}{3}}$$

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(2) **[10 bonus points]** Evaluate

$$I = \int \arccos(x)\, dx \ .$$

$$u = \arccos(x) \ ; \quad dv = dx$$

$$du = \frac{-1}{\sqrt{1-x^2}}\,dx \ ; \quad v = x$$

$$\therefore \ I = \int u\,dv$$

$$= uv - \int v\,du$$

$$= \arccos(x) \cdot x - \int x \left(\frac{-1}{\sqrt{1-x^2}}\right) dx$$

$$= x\arccos(x) - \frac{1}{2} \underbrace{\int \frac{1}{\sqrt{1-x^2}} (-2x)\,dx}_{\substack{\text{let } w = 1-x^2 \\ dw = -2x\,dx}}$$

$$= x\arccos(x) - \frac{1}{2}\int w^{-1/2}\,dw$$

$$= x\arccos(x) - \cancel{\left(\tfrac{1}{2}\right)} \frac{w^{1/2}}{\cancel{\left(\tfrac{1}{2}\right)}} + C$$

$$= x\arccos(x) - (1-x^2)^{1/2} + C.$$