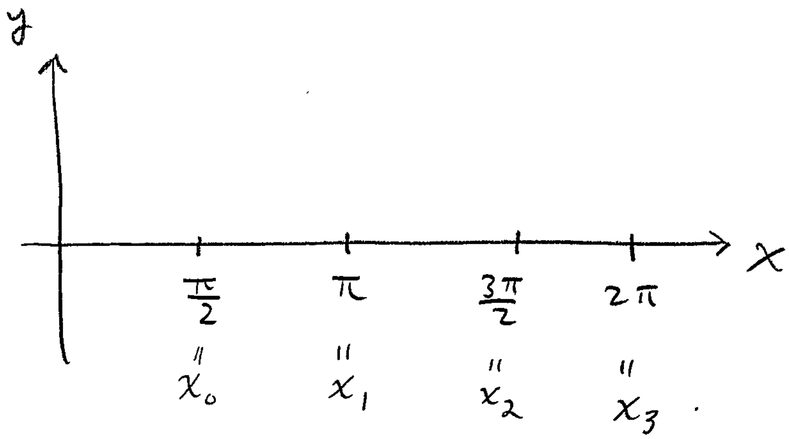
(1) [**7 points**] Use the trapezoid rule with $n = 3$ to approximate $\displaystyle\int_{\pi/2}^{2\pi} \frac{\cos x}{x}\, dx$.



$\Delta x = \frac{\pi}{2}$

$f(x) = \frac{\cos x}{x}$

$$\int_{\frac{\pi}{2}}^{2\pi} \frac{\cos x}{x}\, dx \approx T_3$$

$$= \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + f(x_3) \right]$$

$$= \frac{\left(\frac{\pi}{2}\right)}{2}\left[\cancelto{0}{\frac{\cos\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}}} + 2\frac{\cos(\pi)}{\pi} + \cancelto{0}{\frac{2\cos\left(\frac{3\pi}{2}\right)}{\frac{3\pi}{2}}} + \frac{\cos(2\pi)}{2\pi} \right]$$

$$= \frac{\cancel{\pi}}{4}\left[-\frac{2}{\cancel{\pi}} + \frac{1}{2\cancel{\pi}} \right]$$

$$= \frac{1}{4}\left(\frac{-4+1}{2}\right)$$

$$= \boxed{\frac{-3}{8}}$$

Math 122 F09N01 - Quiz 5 Name: SOLUTIONS.
Nov 12 2009 Stu#:

(2) [**8 points**] Evaluate the improper integral $\int_0^2 z^2 \ln z \, dz$. Clearly and neatly show all details, including any required substitutions or limits.

$$I = \int_0^2 z^2 \ln z \, dz$$

$$= \lim_{b \to 0^+} \int_b^2 z^2 \ln z \, dz$$

For $\int z^2 \ln z \, dz$: $u = \ln z \quad dv = z^2 dz$

$du = \frac{1}{z} dz \quad v = \frac{z^3}{3}.$

$$\therefore \int z^2 \ln z \, dz = uv - \int v \, du$$

$$= \frac{z^3}{3} \ln z - \int \frac{z^3}{3} \frac{1}{z} dz$$

$$= \frac{z^3 \ln z}{3} - \frac{1}{3} \frac{z^3}{3}$$

$$= \frac{z^3 \ln z}{3} - \frac{z^3}{9}.$$

$$\therefore I = \lim_{b \to 0^+} \left[\frac{z^3 \ln z}{3} - \frac{z^3}{9} \right]_b^2$$

$$= \lim_{b \to 0^+} \left[\left(\frac{2^3 \ln 2}{3} - \frac{2^3}{9} \right) - \underbrace{\left(\frac{b^3 \ln b}{3} - \frac{b^3}{9} \right)}_{\to 0} \right]$$

$$= \frac{8}{3} \left(\ln 2 - \frac{1}{3} \right)$$