Math 122 F09N01 - Quiz 4
Oct 23 2009

Name: SOLUTIONS.
Stu#:

(1) [5 points] If $f(1) = 4$, f' is continuous, and $\int_1^4 f'(x)\,dx = 17$, determine the value of $f(4)$.

$$\int_1^4 f'(x)\,dx = 17$$

$$\therefore \quad f(4) - f(1) = 17$$

$$f(4) - 4 = 17$$

$$\therefore \quad \boxed{f(4) = 21}$$

(2) [5 points] Evaluate $\int \frac{1}{\sqrt{1-x^2}\sin^{-1}x}\,dx$.

let $u = \sin^{-1} x$

$du = \frac{1}{\sqrt{1-x^2}}$

$$\therefore \int \frac{1}{\sqrt{1-x^2}\,\sin^{-1}(x)}\,dx = \int \frac{1}{u}\,du$$

$$= \ln|u| + C$$

$$= \boxed{\ln\left|\sin^{-1}(x)\right| + C}$$

(3) [5 points] Compute $\int_0^\pi t\sin(3t)\,dt$.

First : $I = \int t \sin(3t)\,dt$

$$u = t \quad \Big\{ \quad dv = \sin(3t)\,dt$$
$$du = dt \quad \Big| \quad v = -\frac{\cos(3t)}{3}$$

$$\therefore I = \int u\,dv = uv - \int v\,du$$

$$= -\frac{t\cos(3t)}{3} - \int -\frac{\cos(3t)}{3}\,dt$$

$$= -\frac{t\cos(3t)}{3} + \frac{1}{9}\sin(3t) + C$$

$$\therefore \int_0^\pi t\sin(3t)\,dt = \left[-\frac{t\cos(3t)}{3} + \frac{1}{9}\sin(3t)\right]_0^\pi$$

$$= \left[-\frac{\pi \overbrace{\cos(3\pi)}^{-1}}{3} + \cancel{\frac{1}{9}\sin(3\pi)}^{\,0}\right] - \left[\cancel{\frac{-0\cos(0)}{3} + \frac{1}{9}\sin(0)}^{\,0}\right]$$

$$= \boxed{\frac{\pi}{3}}$$