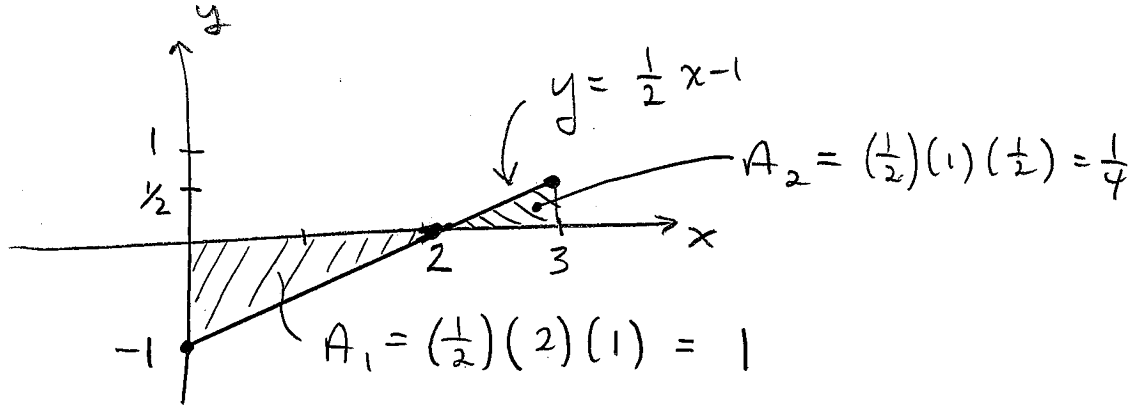
Math 122 F09N01 - Quiz 3 Name: SOLUTIONS.
Oct 9 2009 Stu#:

(1) [2 points] Express $\lim_{n\to\infty} \sum_{i=1}^{n} x_i \sin(x_i)\Delta x$ as a definite integral on the interval $[0, 5]$.

Here : $a = 0$, $b = 5$, $\Delta x = \frac{b-a}{n} = \frac{5}{n}$, $x_i = i\frac{5}{n}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} x_i \sin(x_i)\Delta x = \boxed{\int_0^5 x\sin(x)\,dx.}$$

(2) [5 points] Use an area interpretation to evaluate $\int_0^3 \left(\frac{1}{2}x - 1\right) dx$.



$$\therefore \int_0^3 \left(\frac{1}{2}x - 1\right) dx = A_2 - A_1 = \frac{1}{4} - 1 = \boxed{\frac{-3}{4}}$$

Check: $\int_0^3 \left(\frac{1}{2}x - 1\right) dx = \left[\frac{1}{2}\cdot\frac{x^2}{2} - x\right]_0^3 = \left(\frac{9}{4} - 3\right) - (0) = \frac{9-12}{4} = \frac{-3}{4}.$

(3) **[4 points]** Evaluate $\int_{-2}^{2} (3u+1)^2\, du$.

$$\int_{-2}^{2} (3u+1)^2 du = \int_{-2}^{2} 9u^2+6u+1 \, du$$

$$= \left[9\,\frac{u^3}{3} + \frac{6u^2}{2} + u\right]_{-2}^{2}$$

$$= \left[(3)(2^3)+(3)(2^2)+2\right] - \left[3(-2)^3+3(-2)^2-2\right]$$

$$= 24 \cancel{+12} + 2 + 24 \cancel{-12} + 2$$

$$= \boxed{52}$$

(4) **[4 points]** Evaluate $\int_{0}^{\pi/4} \sec^2 t\, dt$.

$$\int_0^{\frac{\pi}{4}} \sec^2 t\, dt = \Big[\tan t\Big]_0^{\frac{\pi}{4}}$$

$$= \tan\left(\frac{\pi}{4}\right) - \tan(0)$$

$$= 1 - 0$$

$$= \boxed{1} .$$