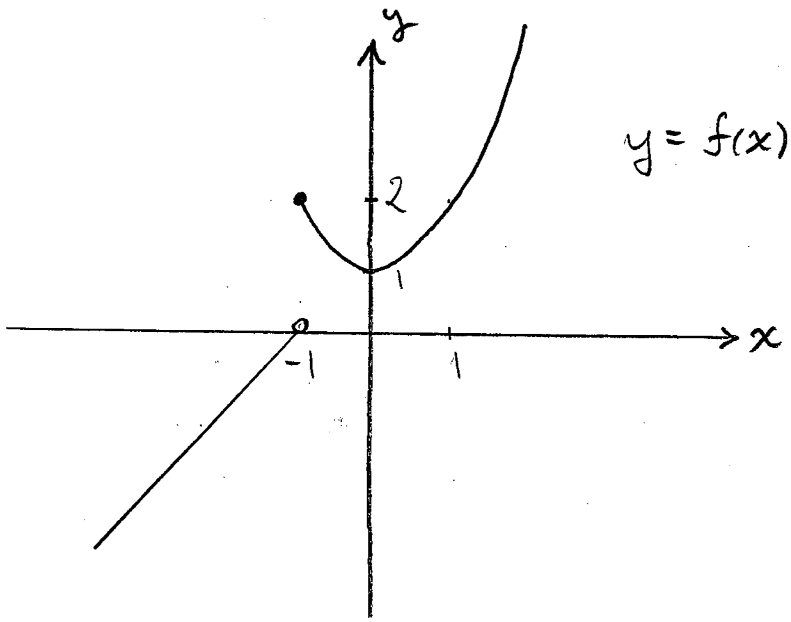
Question 1:

(a)[5 points] Neatly sketch the graph of

$$f(x) = \begin{cases} 1+x & \text{if } x < -1 \\ 1+x^2 & \text{if } x \geq -1 \end{cases}$$



(b)[2 points] Determine both the domain and the range of the function in (a).

Domain : $(-\infty, \infty)$

Range : $(-\infty, 0) \cup [1, \infty)$

(c)[3 points] Use limits to show that ~~g(x)~~ f(x) is not continuous at $x = -1$.

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} 1 + x^2 = 2$$

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} 1 + x = 0$$

Since $\lim_{x \to -1^+} f(x) \neq \lim_{x \to -1^-} f(x)$, f is not continuous at $x = -1$.

Question 2:

(a)[7 points] Let $g(x) = 3x^2 - 5x$. Evaluate and simplify the difference quotient $\dfrac{g(x+h)-g(x)}{h}$.

$$\frac{g(x+h)-g(x)}{h} = \frac{1}{h}\left[3(x+h)^2 - 5(x+h) - 3x^2 + 5x\right]$$

$$= \frac{1}{h}\left[\cancel{3x^2} + 6xh + 3h^2 \cancel{-5x} - 5h \cancel{-3x^2} \cancel{+5x}\right]$$

$$= \frac{1}{\cancel{h}} \cancel{h}(6x + 3h - 5)$$

$$= 6x + 3h - 5$$

(b)[3 points]

Let $F(x) = 1/\sqrt{x + \sqrt{x}}$. If $g(x) = x^2 + x$, determine functions f and h so that $F = f \circ g \circ h$.

$h(x) = \sqrt{x}$ and $f(x) = \dfrac{1}{\sqrt{x}}$.

Then $(f \circ g \circ h)(x) = f(g(h(x)))$

$$= \frac{1}{\sqrt{g(h(x))}}$$

$$= \frac{1}{\sqrt{(h(x))^2 + h(x)}}$$

$$= \frac{1}{\sqrt{x + \sqrt{x}}}$$

Question 3:

(a)[5 points] Evaluate the following limit if it exists: $\lim_{x\to 2} \frac{x^2+2x-8}{x^4-16}$

$$= \lim_{x\to 2} \frac{(x-2)(x+4)}{(x^2-4)(x^2+4)}$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+4)}{\cancel{(x-2)}(x+2)(x^2+4)}$$

$$= \frac{\cancel{6}\ 3}{{}_2\cancel{(4)}(8)}$$

$$= \boxed{\frac{3}{16}}$$

(b)[5 points] Evaluate the following limit if it exists (the Squeeze Theorem may be useful): $\lim_{x\to 0} \sqrt{x^3+x^2}\sin\left(\frac{3}{x}\right)$.

For any $x \geq -1$

$$-\sqrt{x^3+x^2} \leq \sqrt{x^3+x^2}\sin\left(\frac{3}{x}\right) \leq \sqrt{x^3+x^2}$$

Since $\lim_{x\to 0} -\sqrt{x^3+x^2} = 0 = \lim_{x\to 0} \sqrt{x^3+x^2}$,

by the Squeeze Theorem,

$$\lim_{x\to 0} \sqrt{x^3+x^2}\sin\left(\frac{3}{x}\right) = 0$$

Question 4:

(a)[5 points] Evaluate the following limit if it exists: $\lim_{t\to 5} \frac{\sqrt{44+t}-7}{5-t}$

$$\lim_{t\to 5} \frac{\sqrt{44+t}-7}{5-t} \cdot \frac{\sqrt{44+t}+7}{\sqrt{44+t}+7}$$

$$= \lim_{t\to 5} \frac{44+t-49}{(5-t)(\sqrt{44+t}+7)}$$

$$= \lim_{t\to 5} \frac{-5+t}{(5-t)(\sqrt{44+t}+7)}$$

$$= \lim_{t\to 5} \frac{-\cancel{(5-t)}}{\cancel{(5-t)}(\sqrt{44+t}+7)}$$

$$= \boxed{\frac{-1}{14}}$$

(b)[5 points] Evaluate the following limit if it exists: $\lim_{\theta\to 0} \frac{\sin(2\theta)-\theta}{\sin(3\theta)}$

$$\lim_{\theta\to 0} \frac{\sin(2\theta)-\theta}{\sin(3\theta)}$$

$$= \lim_{\theta\to 0} \frac{2\cdot\frac{\sin(2\theta)}{2\theta} - \frac{\theta}{\theta}}{3\cdot\frac{\sin(3\theta)}{3\theta}}$$

$$= \frac{2-1}{3}$$

$$= \boxed{\frac{1}{3}}$$

Question 5:

(a)[**5 points**] Evaluate the following limit if it exists: $\lim_{x\to-\infty} \frac{-2x^3-7x+\pi}{9x^3+11x^2-\pi x+\sqrt{2}}$

$$\lim_{x\to-\infty} \frac{(-2x^3-7x+\pi) \div x^3}{(9x^3+11x^2-\pi x+\sqrt{2}) \div x^3}$$

$$= \lim_{x\to-\infty} \frac{-2-\frac{7}{x^2}+\frac{\pi}{x^3}}{9+\frac{11}{x}-\frac{\pi}{x^2}+\frac{\sqrt{2}}{x^3}}$$

$$= \boxed{\frac{-2}{9}}$$

(b)[**5 points**] Use the Intermediate Value Theorem to show that the equation $2\sin x = \pi - 2x$ has a solution on the interval $[0,\pi]$.

Let $f(x) = 2\sin x - \pi + 2x$.

f is continuous, and $f(0) = -\pi$,

$f(\pi) = 2\sin\pi - \pi + 2\pi = \pi$.

Since $f(0) < 0 < f(\pi)$,

by IVT, there is some $0 < c < \pi$

such that $f(c) = 0$

i.e. $2\sin x - \pi + 2x = 0$ for some $0 < c < \pi$

i.e. $2\sin x = \pi - 2x$ for some $0 < c < \pi$.