Math 121 F09N05 - Quiz 6
Nov 16 2009

Name: SOLUTIONS.
Stu#:

(1) [7 points] If $z^2 = x^2 + y^2$, $dx/dt = 2$, and $dy/dt = 3$, find dz/dt when $x = 5$ and $y = 12$.

$$\frac{d}{dt}\left[z^2\right] = \frac{d}{dt}\left[x^2 + y^2\right]$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

When $x = 5$, $y = 12$, $z = \pm\sqrt{5^2 + 12^2} = \pm 13$.

$$\therefore \quad (2)(\pm 13)\frac{dz}{dt} = (2)(5)(2) + (2)(12)(3)$$

$$\therefore \quad \frac{dz}{dt} = \frac{\cancel{(2)}(5)(2) + \cancel{(2)}(12)(3)}{(2)(\pm 13)}$$

$$= \boxed{\frac{\pm 46}{13}}$$

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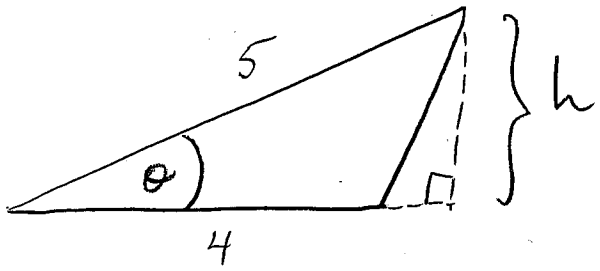
(2) [8 points] Two sides of a triangle are 4 and 5 m in length and the angle between them is increasing at a rate of 3/50 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the two given sides is $\pi/3$.



$h = 5\sin\theta$

$\frac{d\theta}{dt} = \frac{3}{50}\ \frac{\text{rad}}{\text{s}}$.

$$A = \left(\tfrac{1}{2}\right)(4)(h) = (2)(5\sin\theta) = 10\sin\theta.$$

Find $\frac{dA}{dt}$ when $\theta = \frac{\pi}{3}$.

$$\frac{dA}{dt} = \frac{d}{dt}\left[10\sin\theta\right]$$

$$= 10\cos\theta\ \frac{d\theta}{dt}$$

when $\theta = \frac{\pi}{3}$:

$$\frac{dA}{dt} = 10\cos\left(\tfrac{\pi}{3}\right)\cdot\left(\tfrac{3}{50}\right)$$

$$= (10)\left(\tfrac{1}{2}\right)\left(\tfrac{3}{50}\right)$$

$$= \frac{3}{10}\ \frac{m^2}{s}$$

∴ The area is increasing at $\frac{3}{10}\ \frac{m^2}{s}$.