Question 1 [15 points]: Differentiate the following functions (you do not have to simplify your answers, however points will be deducted for improper use of notation):

(a) [**3**] $y = 3x^4 - \frac{\sqrt{x}}{2} - \frac{7}{x}$

(b) [**3**] $f(x) = \tan x \ln x$

(c) [**3**] $g(x) = \frac{3^x}{x + e^x}$

(d) [**3**] $f(x) = e^{\csc(x)}$

(e) [**3**] $y = \cos\left(\sqrt{1 - x^2}\right)$

Question 2 [12 points]:

(a) [4] If $s(t) = t^3 - 3t + 1$ represents the position of a particle in metres at time $t \geq 0$ seconds, determine the acceleration of the particle when the velocity is 9 m/s.

(b) [4] Compute $g''(\pi/4)$ if $g(x) = \ln(\sin x)$.

(c) [4] Determine the point (x, y) on the graph of $y = \dfrac{e^x}{x}$ at which the tangent line is horizontal.

Question 3 [12 points]: Evaluate the following limits (it may be useful to recall that $\lim_{x \to 0} \frac{\sin x}{x} = 1$):

(a) [**3**] $\lim_{x \to -4} \frac{x^2 + x - 12}{x^2 + 2x - 8}$

(b) [**3**] $\lim_{x \to \infty} \frac{-5x^7 + 7x^5}{7x^5 - x - 1}$

(c) [**3**] $\lim_{x \to 0^+} \ln(1 + \sqrt{x})$

(d) [**3**] $\lim_{\theta \to 0} \frac{\sin(3\theta)}{\tan(5\theta)}$

Question 4 [11 points]:

(a) [**3**] Determine the general antiderivative of $f(x) = x^{1/2} - \sec^2 x + \pi$

(b) [**3**] If $f'(x) = 2x - \dfrac{e^x}{2} - 1$ and $f(0) = -1$, determine $f(x)$.

(c) [**5**] A particle has acceleration $a(t) = \sin t + \cos t$ where t is time in seconds. If the initial velocity is $v(0) = -1$ and initial position is $s(0) = 1$, determine the position of the particle at time $t = \pi$ seconds.

Question 5 [12 points]:

(a) [4] Determine the equation of the tangent line to $y = \dfrac{x + \ln x}{x^3}$ at the point where $x = 1$.

(b) [4] At the point where $x = a$ the tangent line to $f(x) = x^3$ is parallel to the tangent line to $g(x) = x^2 + x + 5$. Determine all possible values of a.

(c) [4] Use a linear approximation to estimate $\dfrac{1}{\sqrt{101}}$.

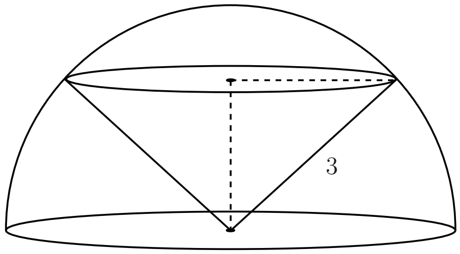
Question 6 [8 points]:

(a) [4] For the curve defined by $\sqrt{xy} = x^2y - 2$, determine the equation of the tangent line at the point $(1, 4)$.

(b) [4] Use logarithmic differentiation to find y' if $y = (\sin x)^{\cos x}$.

Question 7 [8 points]: The volume of a melting cube of ice is decreasing by 2 cm^3/min. At what rate is the surface area of the cube decreasing when the side length of the cube is 4 cm? State units with your answer.

Question 8 [10 points]: A right circular cone is inscribed in the upper half of a sphere of radius 3 m as shown. Find the largest possible volume of such a cone. Clearly justify all conclusions and state units with your answer. (Recall that the volume of a cone is $V = \pi r^2 h/3$.)



Question 9 [10 points]: A box has length equal to twice the width. The cost to ship the box is equal to the sum of the length, width and height. If the box must have a volume of 12 m^3, determine the dimensions (length, width and height) which minimize the shipping cost.

Question 10 [12 points]: For this question consider the function $f(x) = 20x^3 - 3x^5$.

(a) [4] Determine the intervals of increase and decrease of $f(x)$.

(b) [2] State the x-coordinates of the relative extrema of $f(x)$.

(c) [4] Determine the intervals of concavity of $f(x)$.

(d) [2] State the x-coordinates of the inflection points of $f(x)$.