Question 1:

(a)[**3 points**] \$200 is invested at 4% compounded quarterly. What is the value of the investment after 2 years?

$$A = 200\left(1+\frac{0.04}{4}\right)^{(4)(2)} = \$216.57$$

(b)[**3 points**] What is the effective rate of interest equivalent to 5% interest compounded monthly?

$$R = \left(1+\frac{0.05}{12}\right)^{12} - 1 = 5.12\%$$

(c)[**4 points**] What rate of interest compounded semiannually is required to double an investment in 8 years?

$$P\left(1+\frac{i}{2}\right)^{(2)(8)} = 2P$$

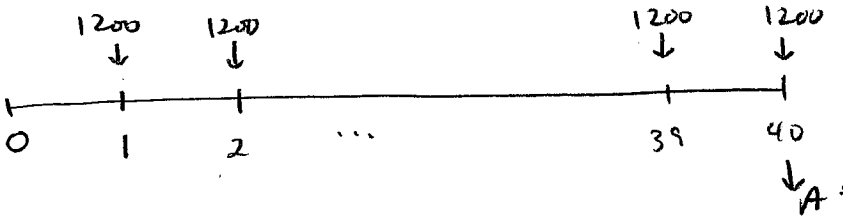
$$1+\frac{i}{2} = 2^{\frac{1}{16}}$$

$$i = 2\left(2^{1/16}-1\right)$$

$$= 8.85\%$$

Question 2:

(a)[5 points] A person deposits \$1200 at the end of each year for 40 years into a retirement fund earning 6% compounded annually. What is the value of the fund at the end of the 40 years?

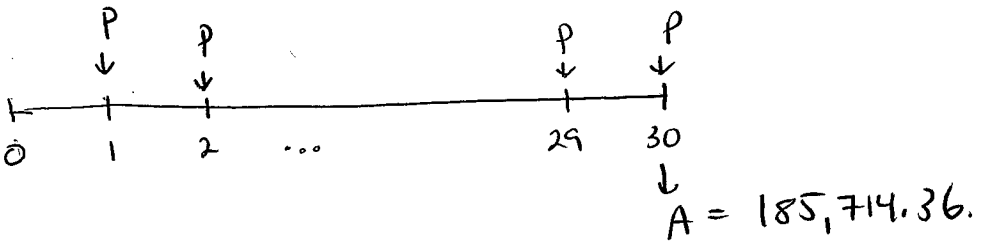


$$A = 1200 + 1200(1.06) + 1200(1.06)^2 + \cdots + 1200(1.06)^{39}$$

$$= 1200\left[\frac{1-(1.06)^{40}}{1-(1.06)}\right]$$

$$= \$185,714.36$$

(b)[5 points] Referring to part (a), suppose the person makes no payments for 10 years and then makes equal payments at the end of each year for 30 years. How large would the payments have to be to accumulate the same fund value by the end of the 40 years?

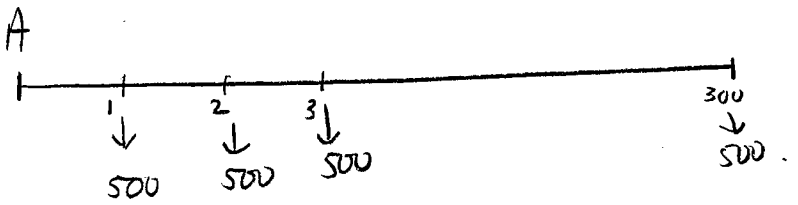


$$\therefore\ 185,714.36 = P\left[\frac{1-(1.06)^{30}}{1-(1.06)}\right]$$

$$\therefore\ P = \frac{(185,714.36)(0.06)}{(1.06)^{30}-1} = \$2349.08$$

Question 3:

(a)[5 points] A person planning retirement would like to have an income of \$500 paid at the end of every month beginning with the first month of retirement. The payments will be made from a fund earning 7% compounded monthly. If the person expects to live for 25 years following retirement, how much must be in the fund on retirement day?

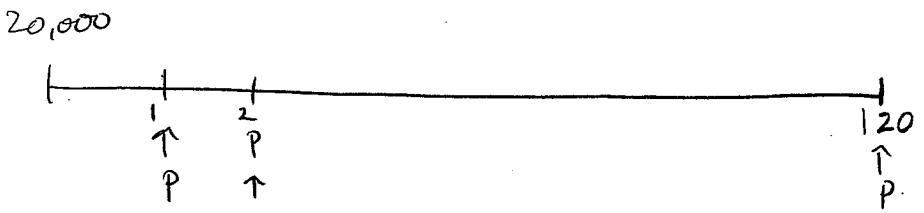


$$A = 500\left(\frac{1}{(1+\frac{0.07}{12})}\right) + 500\left(\frac{1}{1+\frac{0.07}{12}}\right)^2 + \cdots + 500\left(\frac{1}{1+\frac{0.07}{12}}\right)^{300}$$

$$= 500\left(\frac{1}{1+\frac{0.07}{12}}\right)\left[\frac{1-\left(\frac{1}{1+\frac{0.07}{12}}\right)^{300}}{1-\left(\frac{1}{1+\frac{0.07}{12}}\right)}\right]$$

$$= \$70,743.45$$

(b)[5 points] A student has a loan balance of \$20,000 which is to be paid off by making payments at the end of each month for 10 years. The interest rate is 6% compounded monthly. How much are the monthly payments?



$$20000 = P\left(\frac{1}{1+\frac{0.06}{12}}\right) + \cdots + P\left(\frac{1}{1+\frac{0.06}{12}}\right)^{120}$$

$$20,000 = P\left(\frac{1}{1+\frac{0.06}{12}}\right)\left[\frac{1-\left(\frac{1}{1+\frac{0.06}{12}}\right)^{120}}{1-\left(\frac{1}{1+\frac{0.06}{12}}\right)}\right]$$

$$\therefore P = \$222.04$$

Question 4:

(a)[5 points] Let $U = \{1, 2, 3, 4, 5, 6, 7\}$ and

$$A = \{1, 3, 5, 6, \}, \quad B = \{2, 3, 6, 7\}, \quad C = \{4, 6, 7\}$$

Determine

(i) $\bar{B} \cup \bar{A}$ $= \{1, 4, 5\} \cup \{2, 4, 7\} = \{1, 2, 4, 5, 7\}$

(ii) $(A \cap B) \cup \bar{C}$ $= \{3, 6\} \cup \{1, 2, 3, 5\} = \{1, 2, 3, 5, 6\}$

(b)[5 points] If $n(A) = 10$, $n(B) = 8$, $n(A \cap B) = 2$, compute $n(A \cup B)$.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= 10 + 8 - 2$$

$$= 16.$$

Question 5:

(a)[3 points] A club consists of 7 first year students, 9 second year and 8 third year. A committee consisting of 2 first year, 3 second year and 4 third year students is required. How many such committees are possible?

$$C(7,2)\,C(9,3)\,C(8,4)$$

$$= \frac{7!}{5!\,2!} \cdot \frac{9!}{6!\,3!} \cdot \frac{8!}{4!\,4!}$$

$$= 123{,}480$$

(b)[4 points] Mom, Dad and their three children line up for a family photo.

(i) How many different ways can they line up?

$$P(5,5) = 5! = 120$$

(ii) How many different ways can they line up if the three children must be between Mom and Dad?

M _ _ _ D (3! ways) or D _ _ _ M (3! ways.)

$\therefore\ 3! + 3! = 12$ ways.

(c)[3 points] How many distinct ordered arrangements of the letters MÉSSÁGÉ are possible?

$$C(7,1)\,C(6,2)\,C(4,2)\,C(2,1)\,C(1,1)$$

$$= \frac{7!}{1!\,6!} \cdot \frac{6!}{4!\,2!} \cdot \frac{4!}{2!\,2!} \cdot \frac{2!}{1!\,1!} \cdot \frac{1!}{1!\,0!}$$

$$= \frac{7!}{1!\,2!\,2!\,1!\,1!\,0!} = 1260.$$