Question 1 [10 points]: A clothing manufacturer makes three sizes of a particular handmade jacket: small, medium and large. The manufacturing cost per jacket is \$20 for smalls, \$25 for mediums and \$30 for the large. The company has a manufacturing budget of \$20,700 and wants to do a production run of 1000 jackets. Each small jacket requires 6 hours of labour, each medium requires 10 hours of labour, and each large, 20 hours. The total amount of labour available is 6800 hours. Assuming all of the available labour and manufacturing budget is used, how many of each size should be produced?

Clearly label all variables used and state a clear conclusion.

Let x = number of small
y = number of medium
z = number of large.

$$x + y + z = 1000$$
$$20x + 25y + 30z = 20{,}700$$
$$6x + 10y + 20z = 6{,}800$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 20 & 25 & 30 & 20{,}700 \\ 6 & 10 & 20 & 6{,}800 \end{array}\right]$$

$R_2 = (-20)r_1 + r_2$:
$R_3 = (-6)r_1 + r_3$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & 5 & 10 & 700 \\ 0 & 4 & 14 & 800 \end{array}\right]$$

$R_2 = (\frac{1}{5})r_2$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & 1 & 2 & 140 \\ 0 & 4 & 14 & 800 \end{array}\right]$$

$R_3 = (-4)r_2 + r_3$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & 1 & 2 & 140 \\ 0 & 0 & 6 & 240 \end{array}\right]$$

$R_3 = (\frac{1}{6})r_3$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & 1 & 2 & 140 \\ 0 & 0 & 1 & 40 \end{array}\right]$$

$\therefore z = 40$
$y = 140 - 2(40) = 60$
$x = 1000 - 40 - 60 = 900$

$\therefore$ 900 small, 60 medium and 40 large jackets should be produced.

Question 2:

(a)[4 points] Reduce the augmented coefficient matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & -2 & 1 \\ 1 & -2 & 4 & -5 \end{array}\right]$$

to determine how many solutions the system has. (The answer is either zero, exactly one, or infinitely many solutions; which is it)?

$R_3 = (-1)r_1 + r_3$: $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & -2 & 1 \\ 0 & -2 & 4 & -2 \end{array}\right]$

$R_3 = 2r_2 + r_3$: $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\therefore$ system has infinitely many solutions.

(b)[4 points] Let $\mathbf{A} = \begin{bmatrix} 2 & -1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 3 & -5 \\ -9 & 2 \end{bmatrix}$, and $\mathbf{C} = \begin{bmatrix} -2 & 2 \\ 4 & -1 \end{bmatrix}$. Compute $\mathbf{A}(\mathbf{B} + 2\mathbf{C})$.

$B + 2C = \begin{bmatrix} 3 & -5 \\ -9 & 2 \end{bmatrix} + 2\begin{bmatrix} -2 & 2 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -1 & 0 \end{bmatrix}$

$\therefore A(B+2C) = \begin{bmatrix} 2 & -1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -2 \end{bmatrix}$.

(c)[2 points] Suppose $\mathbf{A}$ has dimension 4×1, $\mathbf{B}$ has dimension 6×4, $\mathbf{C}$ has dimension 4×4, and $\mathbf{D}$ has dimension 1×6. What is the dimension of the product $\mathbf{CADB}$?

$$\begin{array}{cccc} C & A & D & B \\ (4\times4) & (4\times1) & (1\times6) & (6\times4) \end{array}$$

$\therefore 4\times4$

$\therefore$ dimension of CADB is 4×4.

Question 3 [10 points]: Find $\mathbf{A}^{-1}$, where $\mathbf{A} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix}$.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 1 & 0 \\ 2 & 3 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_3 = (-2)r_1 + r_3$:

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 1 & 0 \\ 0 & 5 & -2 & -2 & 0 & 1 \end{array}\right]$$

$R_2 = (\frac{1}{2})r_2$:

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 5 & -2 & -2 & 0 & 1 \end{array}\right]$$

$R_1 = r_2 + r_1$:

$R_2 = (-5)r_2 + r_3$:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1/2 & 1 & 1/2 & 0 \\ 0 & 1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & -2 & -\frac{5}{2} & 1 \end{array}\right]$$

$R_3 = 2r_3$:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1/2 & 1 & 1/2 & 0 \\ 0 & 1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -4 & -5 & 2 \end{array}\right]$$

$R_1 = (-\frac{1}{2})r_3 + r_1$:

$R_2 = (\frac{1}{2})r_3 + r_2$:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 3 & -1 \\ 0 & 1 & 0 & -2 & -2 & 1 \\ 0 & 0 & 1 & -4 & -5 & 2 \end{array}\right]$$

$\therefore A^{-1} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$.

Question 4 [10 points]: A company makes two kinds of explosives: low grade and high grade. A maximum of 100 kg of low grade and 150 kg of high grade can be produced each week. One kilogram of low grade requires 60 hours to mix and another 70 hours to package, while a kilogram of the high grade requires 40 hours to mix and 40 hours to package. The mixing department has at most 7200 work hours available each week, while the packaging department has at most 7800. If the profit for one kilogram of low grade is \$60 while that for a kilogram of high grade is \$40, what is the maximum possible profit each week?

Graph paper is provided on the next page. Carefully set up the problem, neatly sketch any required graphs and state a clear conclusion.

Let x = no. of kg of low grade per week,
y = no. of kg of high grade per week,
P = profit

maximize $P = 60x + 40y$

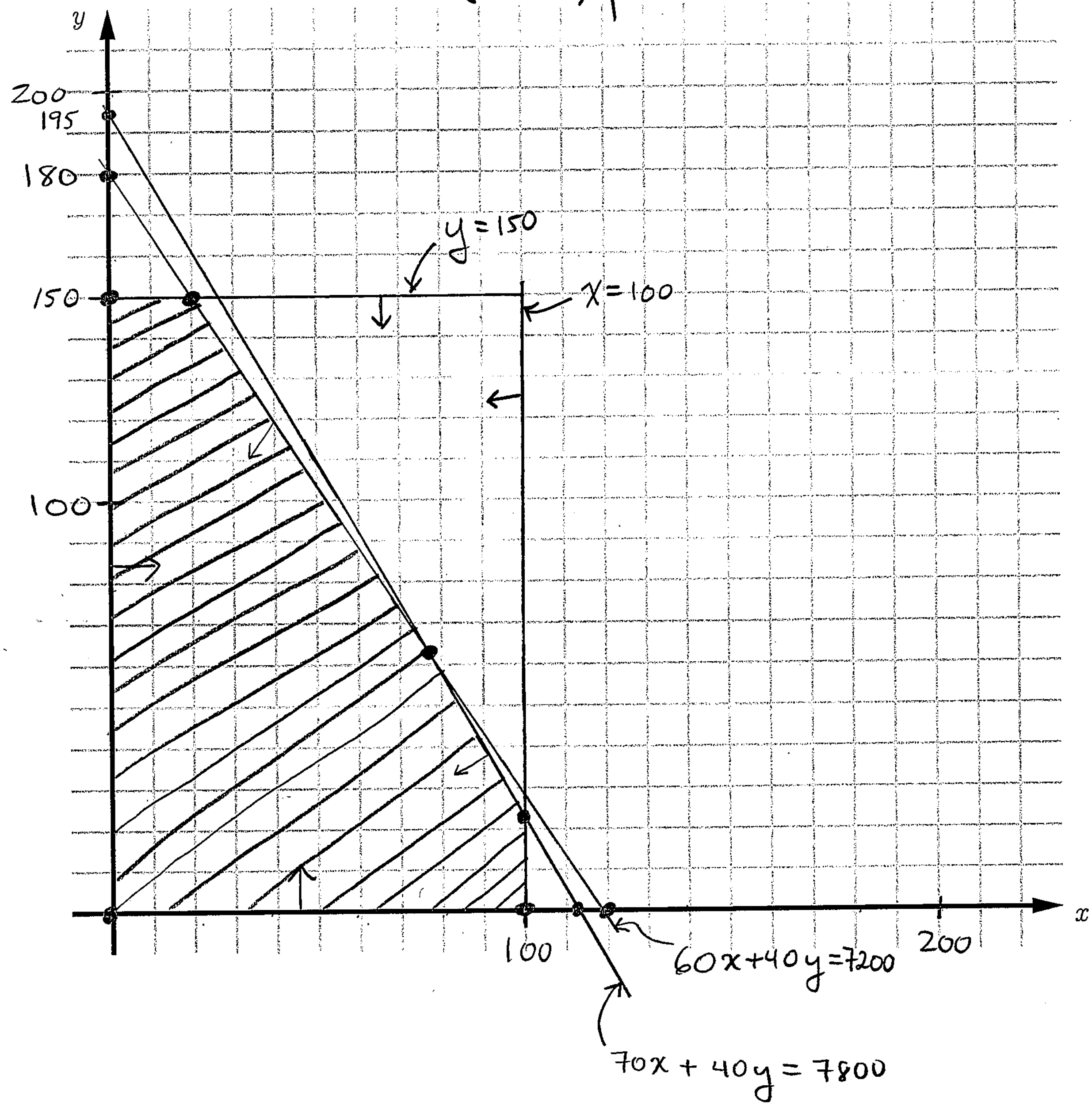
subject to:
$$x \leq 100$$
$$y \leq 150$$
$$60x + 40y \leq 7200$$
$$70x + 40y \leq 7800.$$
$$x \geq 0$$
$$y \geq 0.$$

Corner points:

- By inspection: $(0,0)$, $(100,0)$, $(0,150)$.
- Solving $\left.\begin{array}{l} x = 100 \\ 70x + 40y = 7800 \end{array}\right\}$ $y = \frac{7800 - 70(100)}{40} = 20$, $\therefore (100, 20)$
- Solving $\left.\begin{array}{l} y = 150 \\ 60x + 40y = 7200 \end{array}\right\}$ $x = \frac{7200 - 40(150)}{60} = 20$, $\therefore (20, 150)$
- Solving $\left.\begin{array}{l} ① \; 60x + 40y = 7200 \\ ② \; 70x + 40y = 7800 \end{array}\right\}$ ② − ①: $10x = 600$
  $\therefore x = 60$
  $y = \frac{7200 - 60(60)}{40} = 90$ $\therefore (60, 90)$

Question 4 (continued)

corner pt.	$P = 60x + 40y$
(0,0)	0
(100,0)	6000
(0,150)	6000
(100,20)	6800
(20,150)	7200 } max.
(60,90)	7200 }



∴ maximum profit is \$7200 per week.