

## Question 1:

(a)[3 points] Determine the slope of the line through the points  $(-3, 5)$  and  $(1, -3)$ .

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-3)}{-3 - 1} = \frac{8}{-4} = \boxed{-2}$$

(b)[2 points] Determine an equation of the line through  $(-3, 5)$  and  $(1, -3)$ . You may state your answer using any of the three standard forms of lines we saw.

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -2(x - (-3))$$

$$\boxed{y - 5 = -2(x + 3)} \quad \text{or} \quad \boxed{y = -2x - 1}$$

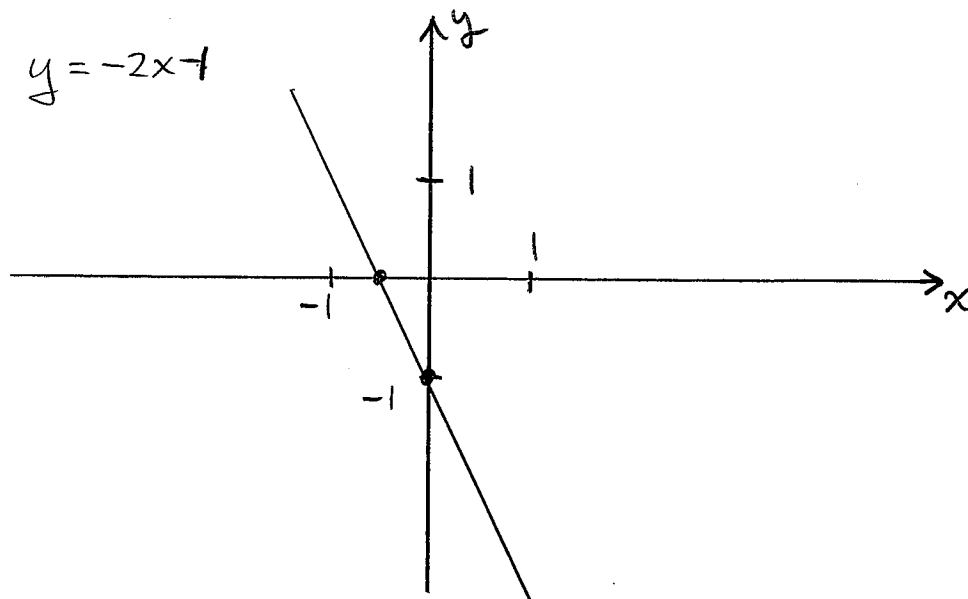
(c)[3 points] Determine the  $x$  and  $y$  intercepts of the line you found in (b).

Using  $y = -2x - 1$ ,  $y$ -intercept is  $(0, -1)$

When  $y = 0$ ,  $-2x - 1 = 0$ , so  $x = -\frac{1}{2}$

$\therefore$  the  $x$ -intercept is  $(-\frac{1}{2}, 0)$ .

(d)[2 points] Neatly graph the line you found in part (b). Label and show the scale on the axes.



## Question 2:

(a) [5 points] Determine the point of intersection of the following pair of lines:

$$3x - 7y = 8 \quad \textcircled{1}$$

$$2x + 4y = -12 \quad \textcircled{2}$$

Using  $\textcircled{2}$  :  $2x = -12 - 4y$   
 $x = -6 - 2y$

sub. into  $\textcircled{1}$  :  $3(-6 - 2y) - 7y = 8$

$$-18 - 6y - 7y = 8$$

$$-13y = 26$$

$$y = -2$$

$$\therefore x = -6 - 2(-2) = -2$$

$$\therefore x = -2, y = -2$$

(b) [5 points] Determine an equation of the line through  $(-1, -2)$  which is parallel to the line  $\frac{2}{3}x + \frac{1}{3}y = 9$ . You may state your equation in any of the standard forms.

$$\frac{2}{3}x + \frac{1}{3}y = 9$$

$$2x + y = 27$$

$$y = -2x + 27$$

$\therefore$  Slope of the line is  $m = -2$ . Using the point  $(-1, -2)$  on the line :

$$y - y_1 = m(x - x_1)$$

$$y + 2 = -2(x + 1)$$

or

$$y = -2x - 4$$

**Question 3 [10 points]:** An investor has \$12,000 to invest and two investments are available. The first investment pays interest at a rate of 5% per year, while the second pays interest at 7% per year. The investor would like to earn a total of \$760 in interest for the year. How much should be invested in each of the investments? Round your answers to the nearest dollar and clearly state your conclusion.

Let  $x$  = amount invested at 5%  
 $y$  = amount invested at 7%.

$$x + y = 12,000 \quad \textcircled{1}$$

$$0.05x + 0.07y = 760 \quad \textcircled{2}$$

using  $\textcircled{1}$ :  $y = 12,000 - x$

Sub. into  $\textcircled{2}$ :  $0.05x + (0.07)(12,000 - x) = 760$

$$0.05x + (0.07)(12,000) - 0.07x = 760$$

$$-0.02x = 760 - (0.07)(12,000)$$

$$x = \frac{760 - (0.07)(12,000)}{-0.02}$$

$$= 4000$$

$$\therefore y = 12,000 - 4,000 = 8,000$$

$\therefore$  \$4,000 should be invested at 5% and  
\$8,000 should be invested at 7%.

**Question 4:** Bart and Lisa set up a lemonade stand and sell the drink for \$0.50 per glass. They calculate that it costs them \$0.20 per glass to produce, but they initially have to spend \$20 to build the stand and another \$7 to make their sign.

(a)[5 points] How many glasses of lemonade must be sold before any profit is realized?

Let  $C = \text{cost}$ ,  $R = \text{revenue}$ ,  $x = \text{number of glasses sold}$ .

$$C = 0.2x + 27$$

$$R = 0.5x$$

Profit is realized once sales surpass the breakeven point:

$$C = R$$

$$0.2x + 27 = 0.5x$$

$$27 = 0.3x$$

$$\therefore x = \frac{27}{0.3} = 90$$

$\therefore$  90 glasses of lemonade must be sold before profit is realized

(b)[5 points] When Bart and Lisa reach \$100 in total sales they realize that they made a mistake: they have just broken even, which means the \$0.20 per glass production cost was wrong. If total revenue of \$100 corresponds to the true break even point, what must be the correct production cost for each glass of lemonade?

$R = \$100$  corresponds to the true break even point.

$$\therefore 100 = 0.5x$$

$\therefore x = 200$  glasses corresponds to the true break even point.

Let  $p = \text{correct production cost per glass}$ .

$$\text{Then } C = p \cdot (200) + 27 = 100$$

$$\therefore p = \frac{100 - 27}{200} = 0.365 \approx 0.37$$

$\therefore$  The production cost per glass is \$0.37

## Question 5:

- (a) [5 points] Solve the following system of equations and state whether the system is consistent or inconsistent. You may solve using any method you wish.

$$3x - y = 2$$

$$-6x + 2y = 4$$

using matrices:

$$\left[ \begin{array}{cc|c} 3 & -1 & 2 \\ -6 & 2 & 4 \end{array} \right]$$

$$R_1 = \frac{1}{3}r_1:$$

$$\left[ \begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{2}{3} \\ -6 & 2 & 4 \end{array} \right]$$

$$R_2 = 6r_1 + r_2:$$

$$\left[ \begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 8 \end{array} \right]$$

$\therefore$  system has no solution ;  
system is inconsistent.

- (b) [5 points] The following system of equations has exactly one solution. Solve the system using matrix reduction:

$$x - 2y + z = -3$$

$$-2y - 2z = -20$$

$$2y + 3z = 27$$

$$\left[ \begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & -2 & -2 & -20 \\ 0 & 2 & 3 & 27 \end{array} \right]$$

$$R_2 = \left(-\frac{1}{2}\right)r_2:$$

$$\left[ \begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & 1 & 1 & 10 \\ 0 & 2 & 3 & 27 \end{array} \right]$$

$$R_3 = (-2)r_2 + r_3:$$

$$\left[ \begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & 1 & 1 & 10 \\ 0 & 0 & 1 & 7 \end{array} \right]$$

$$\therefore z = 7$$

$$y + z = 10 \Rightarrow y = 10 - z = 10 - 7 = 3$$

$$\begin{aligned} x - 2y + z = -3 &\Rightarrow x = -3 + 2y - z \\ &= -3 + 2(3) - 7 \\ &= -4 \end{aligned}$$

$$\therefore x = -4, y = 3, z = 7$$