Math 111 F09N02 - Quiz 3
Oct 9 2009

Name: SOLUTIONS.
Stu#:

(1) [**8 points**] A juice company produces orange, tomato and pineapple juice. Each case of orange juice requires 10 minutes cleaning, 4 minutes filling and 2 minutes for labeling. Each case of tomato juice requires 12 minutes cleaning, 4 minutes filling and 1 minute for labeling. Each case of pineapple juice requires 9 minutes cleaning, 6 minutes filling and 1 minute for labeling. If the cleaning machine runs for 398 minutes, the filling machine for 164 minutes and the labeling machine for 58 minutes, how many cases of each type of juice were produced?

Let x = no. of cases of orange
y = no. of tomato
z = no. of pineapple.

Cleaning: $10x + 12y + 9z = 398$
filling: $4x + 4y + 6z = 164$
labeling: $2x + y + z = 58$

$$\left[\begin{array}{ccc|c} 10 & 12 & 9 & 398 \\ 4 & 4 & 6 & 164 \\ 2 & 1 & 1 & 58 \end{array}\right]$$

$r_1 \leftrightarrow r_2$:

$$\left[\begin{array}{ccc|c} 4 & 4 & 6 & 164 \\ 10 & 12 & 9 & 398 \\ 2 & 1 & 1 & 58 \end{array}\right]$$

$R_1 = \frac{1}{4} r_1$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 3/2 & 41 \\ 10 & 12 & 9 & 398 \\ 2 & 1 & 1 & 58 \end{array}\right]$$

$R_2 = (-10) r_1 + r_2$:
$R_3 = (-2) r_1 + r_3$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 3/2 & 41 \\ 0 & 2 & -6 & -12 \\ 0 & -1 & -2 & -24 \end{array}\right]$$

$R_2 = \frac{1}{2} r_2$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 3/2 & 41 \\ 0 & 1 & -3 & -6 \\ 0 & -1 & -2 & -24 \end{array}\right]$$

$R_3 = r_3 + r_2$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 3/2 & 41 \\ 0 & 1 & -3 & -6 \\ 0 & 0 & -5 & -30 \end{array}\right]$$

$R_3 = (-\frac{1}{5}) r_3$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 3/2 & 41 \\ 0 & 1 & -3 & -6 \\ 0 & 0 & 1 & 6 \end{array}\right]$$

$\therefore z = 6$,
$y - 3z = -6 \Rightarrow y = -6 + 3(6) = 12$
$x + y + (3/2)z = 41 \Rightarrow x = 41 - 12 - (\frac{3}{2})(6) = 20$

$\therefore$ 20 cases of orange, 12 cases tomato and 6 cases of pineapple should be produced

(2) [**3 points**] Let

$$\mathbf{A} = \begin{bmatrix} 2 & -3 & 4 \\ 0 & 2 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & -2 & 0 \\ 5 & 1 & 2 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} -3 & 0 & 5 \\ 2 & 1 & 3 \end{bmatrix}$$

Compute $2(\mathbf{A} - \mathbf{B}) - \mathbf{C}$.

$$2(A-B)-C = 2\left(\begin{bmatrix} 2 & -3 & 4 \\ 0 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -2 & 0 \\ 5 & 1 & 2 \end{bmatrix}\right) - \begin{bmatrix} -3 & 0 & 5 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= 2\begin{bmatrix} 1 & -1 & 4 \\ -5 & 1 & -1 \end{bmatrix} - \begin{bmatrix} -3 & 0 & 5 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & 8 \\ -10 & 2 & -2 \end{bmatrix} - \begin{bmatrix} -3 & 0 & 5 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -2 & 3 \\ -12 & 1 & -5 \end{bmatrix}$$

(3) [**4 points**] Determine x and y if

$$\begin{bmatrix} x-2y & 0 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ -2 & x+y \end{bmatrix}$$

① $x - 2y = 3$
② $x + y = 6.$

using ①: $x = 3 + 2y$

sub. into ②: $3 + 2y + y = 6$

$3y = 3$

$y = 1$

$\therefore x = 3 + 2y$
$= 3 + 2(1)$
$= 5$

$$\boxed{\therefore x = 5,\ y = 1}$$