Question 1:

(a)[3 points]   Determine the slope of the line through the points $(-3, 5)$ and $(1, -3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-3)}{-3 - 1} = \frac{8}{-4} = \boxed{-2}$$

(b)[2 points]   Determine an equation of the line through $(-3, 5)$ and $(1, -3)$. You may state your answer using any of the three standard forms of lines we saw.

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -2(x - (-3))$$

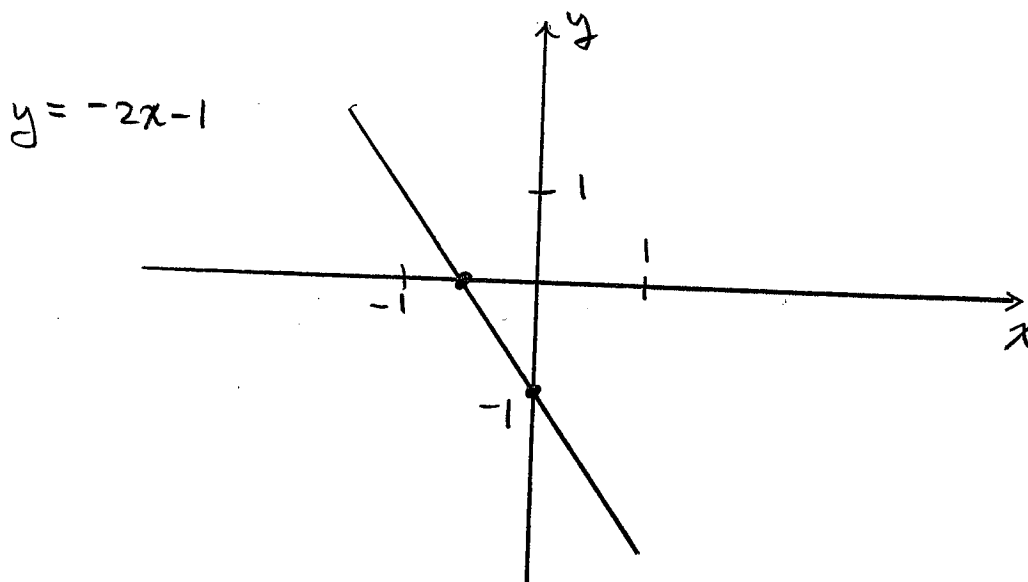$$\boxed{y - 5 = -2(x+3)} \quad \text{or} \quad \boxed{y = -2x - 1}$$

(c)[3 points]   Determine the x and y intercepts of the line you found in (b).

Using $y = -2x - 1$, y-intercept is $(0, -1)$.

When $y = 0$, $-2x - 1 = 0$,   so $x = \frac{-1}{2}$.

$\therefore$ x intercept is $\left(\frac{-1}{2}, 0\right)$

(d)[2 points]   Neatly graph the line you found in part (b). Label and show the scale on the axes.

$y = -2x - 1$

Question 2:

(a)[5 points]   Determine the point of intersection of the following pair of lines:

$$3x - 7y = 8 \quad ①$$
$$2x + 4y = -12 \quad ②$$

Using ② : $2x = -12 - 4y$

$x = -6 - 2y$

sub - into ① : $3(-6 - 2y) - 7y = 8$

$-18 - 6y - 7y = 8$

$-13y = 26$

$y = -2$

$\therefore x = -6 - 2(-2) = -2$

$$\boxed{\therefore \; x = -2, \; y = -2}$$

(b)[5 points]   Determine an equation of the line through $(-1, -2)$ which is parallel to the line $\frac{2}{3}x + \frac{1}{3}y = 9$. You may state your equation in any of the standard forms.

$\frac{2}{3}x + \frac{1}{3}y = 9$

$2x + y = 27$

$y = -2x + 27$

$\therefore$ slope of line is $m = -2$, using point $(-1, -2)$ on the line :

$y - y_1 = m(x - x_1)$

$$\boxed{y + 2 = -2(x+1)} \quad \underset{=}{\text{or}} \quad \boxed{y = -2x - 4}$$

Question 3 [10 points]: An investor has \$12,000 to invest and two investments are available. The first investment pays interest at a rate of 5% per year, while the second pays interest at 7% per year. The investor would like to earn a total of \$760 in interest for the year. How much should be invested in each of the investments? Round your answers to the nearest dollar and clearly state your conclusion.

Let $\quad x$ = amount invested at 5%

$\quad\quad\quad y$ = amount invested at 7%.

$$x + y = 12,000 \quad\text{①}$$

$$0.05x + 0.07y = 760 \quad\text{②}$$

using ① : $\quad y = 12,000 - x$

sub. into ② :

$$0.05x + 0.07(12,000 - x) = 760$$

$$0.05x + (0.07)(12,000) - 0.07x = 760$$

$$-0.02x = 760 - (0.07)(12,000)$$

$$\therefore \quad x = \frac{760 - (0.07)(12,000)}{-0.02}$$

$$= 4000.$$

$$\therefore \quad y = 12,000 - 4000 = 8000.$$

$\therefore$ \$4,000 should be invested at 5% and \$8,000 should be invested at 7%.

Question 4: Bart and Lisa set up a lemonade stand and sell the drink for $0.50 per glass. They calculate that it costs them $0.20 per glass to produce, but they initially have to spend $20 to build the stand and another $7 to make their sign.

(a)[5 points] How many glasses of lemonade must be sold before any profit is realized?

Let C = cost, R = revenue, x = number of glasses sold.

$C = 0.2x + 27$.

$R = 0.5x$.

Profit is realized once sales surpass the break even point:

$$C = R$$

$$0.2x + 27 = 0.5x$$

$$27 = 0.3x$$

$$\therefore x = \frac{27}{0.3} = 90$$

$\therefore$ 90 glasses of lemonade must be sold before profit is realized.

(b)[5 points] When Bart and Lisa reach $100 in total sales they realize that they made a mistake: they have just broken even, which means the $0.20 per glass production cost was wrong. If total revenue of $100 corresponds to the true break even point, what must be the correct production cost for each glass of lemonade?

$R = \$100$ corresponds to the true break even point.

$\therefore 100 = 0.5x$

$\therefore x = 200$ glasses corresponds to the true break even pt.

Let p = true production cost per glass.

$\therefore C = px + 27 = 100$

$\therefore p(200) + 27 = 100$

$\therefore p = \frac{100 - 27}{200} = 0.365 \approx 0.37$.

$\therefore$ The production cost per glass is $\$0.37$

Question 5:

(a)[**5 points**]   Solve the following system of equations and state whether the system is consistent or inconsistent. You may solve using any method you wish.

$$3x - y = 2$$
$$-6x + 2y = 4$$

using matrices:

$$\begin{bmatrix} 3 & -1 & | & 2 \\ -6 & 2 & | & 4 \end{bmatrix}$$

$R_1 = \frac{1}{3} r_1:$ $\begin{bmatrix} 1 & -\frac{1}{3} & | & \frac{2}{3} \\ -6 & 2 & | & 4 \end{bmatrix}$

$R_2 = 6r_1 + r_2:$

$$\begin{bmatrix} 1 & -\frac{1}{3} & | & \frac{2}{3} \\ 0 & 0 & | & 8 \end{bmatrix}$$

∴ system has <u>no solution</u>; the system is <u>inconsistent</u>.

(b)[**5 points**]   The following system of equations has exactly one solution. Solve the system using matrix reduction:

$$x - 2y + z = -3$$
$$-2y - 2z = -20$$
$$2y + 3z = 27$$

$$\begin{bmatrix} 1 & -2 & 1 & | & -3 \\ 0 & -2 & -2 & | & -20 \\ 0 & 2 & 3 & | & 27 \end{bmatrix}$$

$R_2 = \left(\frac{-1}{2}\right) r_2:$

$$\begin{bmatrix} 1 & -2 & 1 & | & -3 \\ 0 & 1 & 1 & | & 10 \\ 0 & 2 & 3 & | & 27 \end{bmatrix}$$

$R_3 = (-2) r_2 + r_3:$

$$\begin{bmatrix} 1 & -2 & 1 & | & -3 \\ 0 & 1 & 1 & | & 10 \\ 0 & 0 & 1 & | & 7 \end{bmatrix}$$

∴ $z = 7$

$y + z = 10 \Rightarrow y = 10 - z = 10 - 7 = 3$

$x - 2y + z = -3 \Rightarrow x = -3 + 2y - z$
$$= -3 + 2(3) - 7$$
$$= -4$$

∴ $x = -4, \ y = 3, \ z = 7$