

(1) [8 points] You have \$10,000 to invest and three investments available to you: one pays 6%, the second 7% and the third pays 8% annually. If you wish to receive \$680 per year from your investments and you require that the amount invested at 8% be half the amount invested at 6%, how much should you place in each investment?

Let  $x$  = amount invested at 6%  
 $y$  = amount invested at 7%  
 $z$  = amount invested at 8%.

$$\left. \begin{aligned} x+y+z &= 10,000 \\ 0.06x+0.07y+0.08z &= 680 \\ z &= \frac{1}{2}x \end{aligned} \right\} \begin{aligned} \therefore x+y+z &= 10,000 \\ (0.06)x+(0.07)y+(0.08)z &= 680 \\ \frac{1}{2}x &+ z = 0 \end{aligned}$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 10,000 \\ 0.06 & 0.07 & 0.08 & 680 \\ \frac{1}{2} & 0 & -1 & 0 \end{array} \right]$$

$R_2 = 100r_2$  ;  
 $R_3 = 2r_3$  ;

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 10,000 \\ 6 & 7 & 8 & 68,000 \\ 1 & 0 & -2 & 0 \end{array} \right]$$

$R_2 = (-6)r_1 + r_2$  ;  
 $R_3 = (-1)r_1 + r_3$  ;

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 10,000 \\ 0 & 1 & 2 & 8,000 \\ 0 & -1 & -3 & -10,000 \end{array} \right]$$

$R_3 = r_2 + r_3$  ;

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 10,000 \\ 0 & 1 & 2 & 8,000 \\ 0 & 0 & -1 & -2,000 \end{array} \right]$$

$R_3 = (-1)r_3$  ;

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 10,000 \\ 0 & 1 & 2 & 8,000 \\ 0 & 0 & 1 & 2,000 \end{array} \right]$$

$\therefore z = \$2,000$  ,

$y + 2z = 8000 \Rightarrow y = 8000 - 2(2000) = \$4,000$

$x + y + z = 10000 \Rightarrow x = 10000 - 2000 - 4000 = 4000$

$\therefore$  \$4,000 should be invested at 6%,  
 \$4,000 at 7%, and \$2,000 at 8%.

(2) [3 points] Let

$$A = \begin{bmatrix} 2 & -3 & 4 \\ 0 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -2 & 0 \\ 5 & 1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & 0 & 5 \\ 2 & 1 & 3 \end{bmatrix}$$

Compute  $2(A - B) - C$ .

$$\begin{aligned} 2(A - B) - C &= 2 \left( \begin{bmatrix} 2 & -3 & 4 \\ 0 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -2 & 0 \\ 5 & 1 & 2 \end{bmatrix} \right) - \begin{bmatrix} -3 & 0 & 5 \\ 2 & 1 & 3 \end{bmatrix} \\ &= 2 \begin{bmatrix} 1 & -1 & 4 \\ -5 & 1 & -1 \end{bmatrix} - \begin{bmatrix} -3 & 0 & 5 \\ 2 & 1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -2 & 8 \\ -10 & 2 & -2 \end{bmatrix} - \begin{bmatrix} -3 & 0 & 5 \\ 2 & 1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 5 & -2 & 3 \\ -12 & 1 & -5 \end{bmatrix}. \end{aligned}$$

(3) [4 points] Determine  $x$  and  $y$  if

$$\begin{bmatrix} x - 2y & 0 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ -2 & x + y \end{bmatrix}$$

$$\textcircled{1} \quad x - 2y = 3$$

$$\textcircled{2} \quad x + y = 6$$

using  $\textcircled{1}$  :  $x = 3 + 2y$

sub. into  $\textcircled{2}$ :  $3 + 2y + y = 6$

$$3y = 3$$

$$y = 1$$

$$\begin{aligned} \therefore x &= 3 + 2y \\ &= 3 + 2(1) \\ &= 5 \end{aligned}$$

$$\therefore x = 5, y = 1$$