Question 1:

(a)[3 points] Expand and simplify:

$$(x^2 - x)(2x^5 + 3x - 7)$$

$$= 2x^7 + 3x^3 - 7x^2 - 2x^6 - 3x^2 + 7x$$

$$= 2x^7 - 2x^6 + 3x^3 - 10x^2 + 7x$$

(b)[3 points] Simplify and express your answer so that all exponents are positive:

$$\frac{3x^{-2}(yz)^2}{(xy)^{-1}z^3}$$

$$= \frac{3x^{-2}y^2z^2}{x^{-1}y^{-1}z^3}$$

$$= \frac{3y^3}{xz}$$

(c)[4 points] Find the remainder when $5x^4 - 3x^2 + x - 2$ is divided by $x^2 + 1$.

$$\begin{array}{r} 5x^2 - 8 \\ x^2 + 0x + 1 \overline{)\, 5x^4 + 0x^3 - 3x^2 + x - 2} \\ -(5x^4 + 0x^3 + 5x^2) \\ \hline -8x^2 + x - 2 \\ -(-8x^2 + 0x - 8) \\ \hline x + 6 \end{array}$$

$\therefore r(x) = x + 6$

Question 2:

(a)[3 points] Factor completely

$$x^2 - 4x - 12$$

$$= (x-6)(x+2)$$

(b)[3 points] Factor completely:

$$6x^2 - x - 2$$

$$= 6x^2 + 3x - 4x - 2$$

$$= 3x(2x+1) - 2(2x+1)$$

$$= (3x-2)(2x+1)$$

(c)[4 points] Factor completely:

$$x^3 - 3x^2 - x + 3$$

$$= x^2(x-3) - (x-3)$$

$$= (x-3)(x^2-1)$$

$$= (x-3)(x-1)(x+1)$$

Question 3:

(a)[4 points] Simplify:

$$\frac{3}{x-2} - \frac{15}{x^2+x-6}$$

$$= \frac{3}{x-2} - \frac{15}{(x-2)(x+3)}$$

$$= \frac{3(x+3) - 15}{(x-2)(x+3)}$$

$$= \frac{3x+9-15}{(x-2)(x+3)}$$

$$= \frac{3(x-2)}{(x-2)(x+3)} = \frac{3}{x+3}$$

(b)[3 points] Solve:

$$x + \frac{2}{x-2} = 5$$

$$x(x-2) + 2 = 5(x-2) \qquad (x \neq 2)$$

$$x^2 - 2x + 2 = 5x - 10$$

$$x^2 - 7x + 12 = 0$$

$$(x-4)(x-3) = 0$$

$$x = 4, \quad x = 3.$$

(c)[3 points] Simplify:

$$\frac{\frac{3-x}{3+x}}{\frac{3x}{x^2-9}}$$

$$= \frac{3-x}{3+x} \cdot \frac{x^2-9}{3x}$$

$$= \frac{-(x-3)}{\cancel{(x+3)}} \cdot \frac{(x-3)\cancel{(x+3)}}{3x}$$

$$= -\frac{(x-3)^2}{3x}.$$

Question 4:

(a)[3 points] Solve:

$$x^2 - x - 10 = x + 5$$

$$x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

$$x = 5 \quad , \quad x = -3$$

(b)[4 points] Solve:

$$2x^2 - 3x + \frac{1}{2} = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(\frac{1}{2})}}{2(2)}$$

$$= \frac{3 \pm \sqrt{5}}{4}$$

$$= \frac{3+\sqrt{5}}{4} \, , \, \frac{3-\sqrt{5}}{4} .$$

(c)[3 points] Solve and state your answer using interval notation:

$$7 - \frac{1}{3}(x - 5) \leq 1$$

$$-\frac{1}{3}(x-5) \leq -6$$

$$x - 5 \geq 18$$

$$x \geq 23$$

$$\therefore [23, \infty)$$

Question 5:

(a)[3 points] Solve:

$$1 < 2 - \frac{1}{3}x < 5$$

$$-1 < \frac{-1}{3}x < 3$$

$$-3 < -x < 9$$

$$-9 < x < 3$$

$$\therefore \quad (-9, 3)$$

(b)[3 points] Solve and state your answer using interval notation:

$$\left|\frac{2x-6}{7}\right| \leq 1$$

$$-1 \leq \frac{2x-6}{7} \leq 1$$

$$-7 \leq 2x-6 \leq 7$$

$$-1 \leq 2x \leq 13$$

$$\frac{-1}{2} \leq x \leq \frac{13}{2}$$

$$\therefore \quad \left[\frac{-1}{2}, \frac{13}{2}\right]$$

(c)[3 points] Rationalize the denominator:

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} \cdot \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$$

$$= \frac{a + 2\sqrt{ab} + b}{a - b}$$

Question 6:

(a)[5 points] The distance between the points $(-2, 3)$ and $(1, a+3)$ is 4. Find all possible values of a.

$$4 = \sqrt{(-2-1)^2 + (3-(a+3))^2}$$

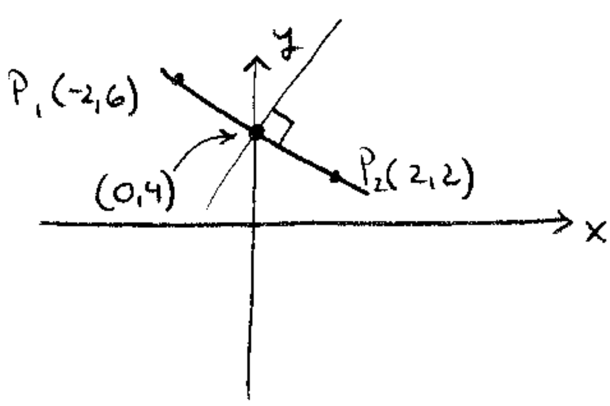
$$4 = \sqrt{9 + a^2}$$

$$16 = 9 + a^2$$

$$a^2 = 7$$

$$a = \sqrt{7},\ -\sqrt{7}.$$

(b)[5 points] A line is perpendicular to the line through $(-2, 6)$ and $(2, 2)$ and passes through the midpoint of these two points. Find the equation of the line.



$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{-2-2} = \frac{4}{-4} = -1 \quad \Big\} \quad \therefore \text{ slope of our line is } m_2 = \frac{-1}{m_1} = \frac{-1}{-1} = 1$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2+2}{2}, \frac{6+2}{2}\right) = (0, 4)$$

$$\therefore \quad y = mx + b$$

$$\boxed{y = x + 4}$$

Question 7:

(a)[3 points] Find the y-intercepts of the graph of

$$y^2 - y = x^3 - x + 12$$

$$y^2 - y = 12$$

$$y^2 - y - 12 = 0$$

$$(y+3)(y-4) = 0$$

$$y = -3, \quad y = 4$$

(b)[3 points] The line through $(0, a)$ and (a, b) has slope $m = -1$. Determine the value of b.

$$-1 = \frac{b-a}{a-0}$$

$$-1 = \frac{b-a}{a}$$

$$-a = b - a$$

$$\boxed{0 = b}$$

(c)[4 points] Determine the radius of the circle:

$$x^2 + y^2 - 8x + 3y = \frac{3}{4}$$

$$(x-4)^2 - 16 + \left(y + \frac{3}{2}\right)^2 - \frac{9}{4} = \frac{3}{4}$$

$$(x-4)^2 + \left(y + \frac{3}{2}\right)^2 = 19$$

$$\therefore \; r = \sqrt{19}.$$

Question 8:

(a)[4 points] Find the domain of the function $f(x) = \dfrac{x}{(x-2)\sqrt{x+3}}$

Must have (i) $x+3>0 \Rightarrow x>-3$

(ii) $x-2\neq 0 \Rightarrow x\neq 2$

$\therefore\ (-3,2)\cup(2,\infty)$

(b)[4 points] Let $f(x) = x^2 - 3x$. Compute and simplify

$$\frac{f(x+h)-f(x)}{h}$$

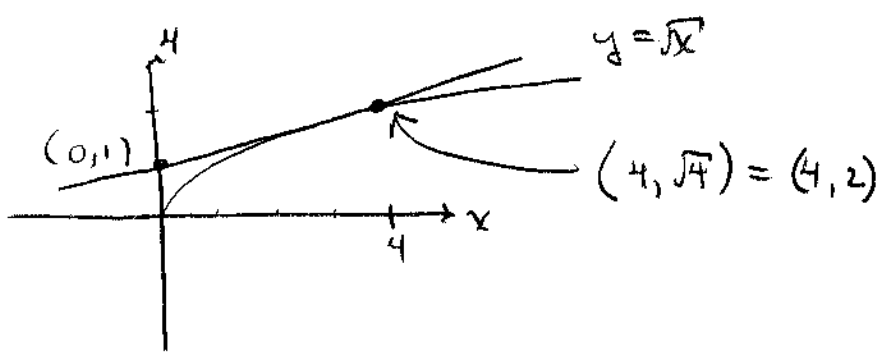
$$= \frac{(x+h)^2 - 3(x+h) - x^2 + 3x}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h}$$

$$= \frac{h(2x+h-3)}{h}$$

$$= 2x + h - 3$$

(c)[4 points] Let $f(x) = \sqrt{x}$. Find the equation of line through $(0,1)$ and the point on the graph of f with x-coordinate $x = 4$.



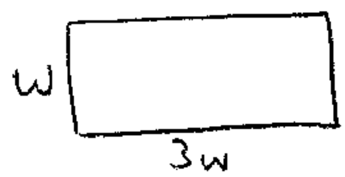
$\therefore\ m = \dfrac{2-1}{4-0} = \dfrac{1}{4}$

$\therefore\ y = mx + b$

$y = \frac{1}{4}x + 1$

Question 9:

(a)[4 points] A rectangle has length equal to three times the width. If x represents the perimeter of the rectangle, find a formula for $A(x)$, the area of the rectangle as a function of the perimeter x.



$$2[w+3w] = x$$
$$8w = x$$
$$w = \frac{x}{8}$$

$$A = (w)(3w) = 3w^2$$

$$\therefore A(x) = 3\left(\frac{x}{8}\right)^2 = \frac{3x^2}{64}$$

(b)[3 points] Determine the vertex of the parabola

$$y = x^2 - 3x - \frac{3}{4}.$$

$$= \left(x-\frac{3}{2}\right)^2 - \frac{9}{4} - \frac{3}{4}$$

$$= \left(x-\frac{3}{2}\right)^2 - 3$$

$\therefore$ vertex is $\left(\frac{3}{2}, -3\right)$

(c)[4 points] The parabola $y = ax^2 + bx + c$ has vertex $(0, 1)$ and passes through the point $(1, 3)$. Determine a, b and c.

Since parabola passes through $(0,1)$, $1 = a\cdot 0^2 + b\cdot 0 + c$

$\therefore \boxed{c=1}$

By symmetry, parabola passes through both $(1,3)$ & $(-1,3)$,

$\therefore 3 = a\cdot 1^2 + b\cdot 1 + 1$

$3 = a(-1)^2 + b(-1) + 1$

$\therefore 2 = a + b$

$2 = a - b \Rightarrow a = 2 + b$

$\therefore 2 = (2+b) + b$

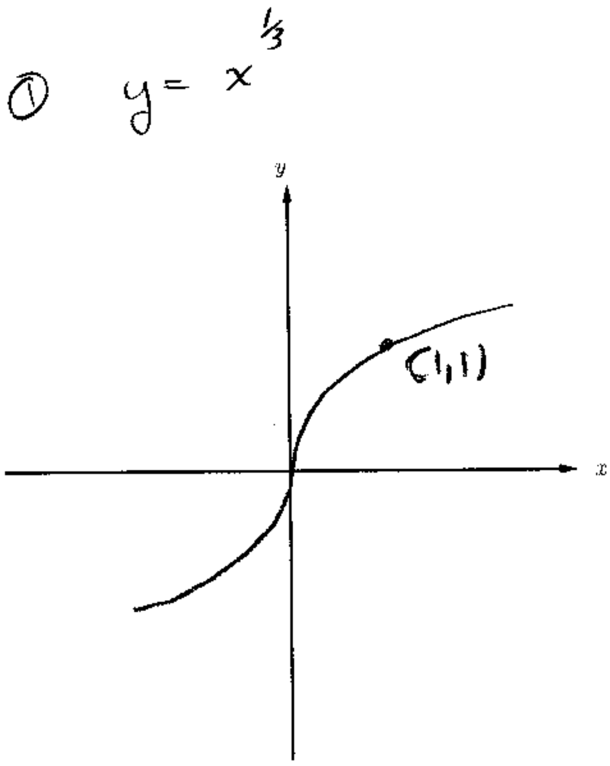
$\therefore 0 = 2b$

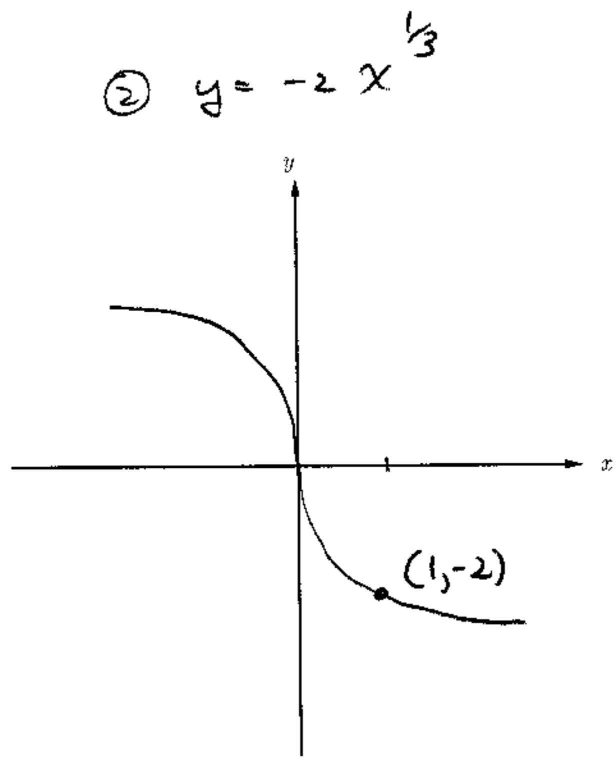
$\therefore b = 0$

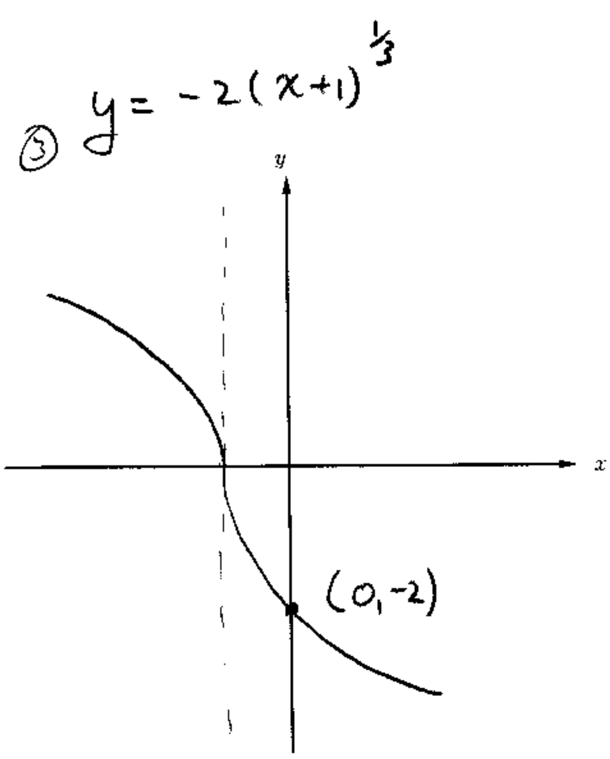
$\therefore a = 2 + b = 2 + 0 = 2$

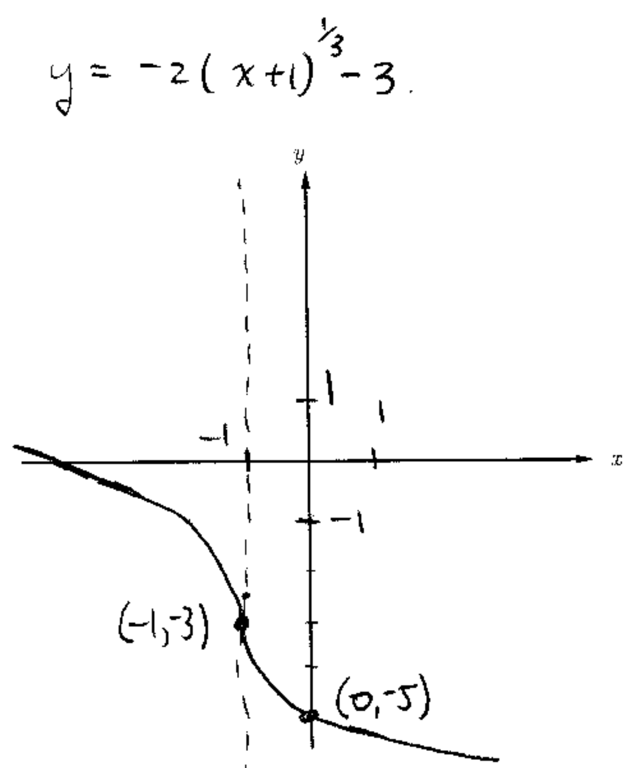
$\therefore a = 2,\ b = 0,\ c = 1$

Question 10 [10 points]: Neatly sketch the graph of the function $f(x) = -2(x+1)^{1/3} - 3$ below by starting with a basic function and applying three transformations. Your final answer should appear in the last graph below. In your final graph indicate the scale on the x and y axes and label at least one point.









Question 11 [8 points]: Let $R(x) = 8x$ and $C(x) = 120+2x$ be the revenue and cost, respectively, resulting from the manufacture and sale of x units per day. Determine the revenue at the *break even point*, the point at which profit equals zero. (Recall that profit is revenue minus cost.)

At break even,

$$R(x) - C(x) = 0$$

$$\therefore 8x - (120+2x) = 0$$

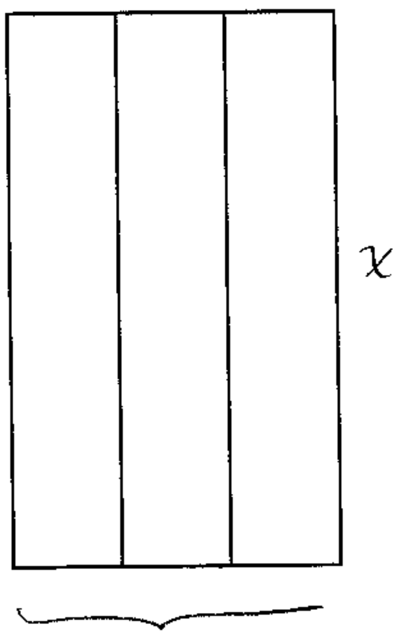
$$8x - 120 - 2x = 0$$

$$6x - 120 = 0$$

$$x = \frac{120}{6} = 20.$$

$$\therefore R(20) = 8(20) = \$160.$$

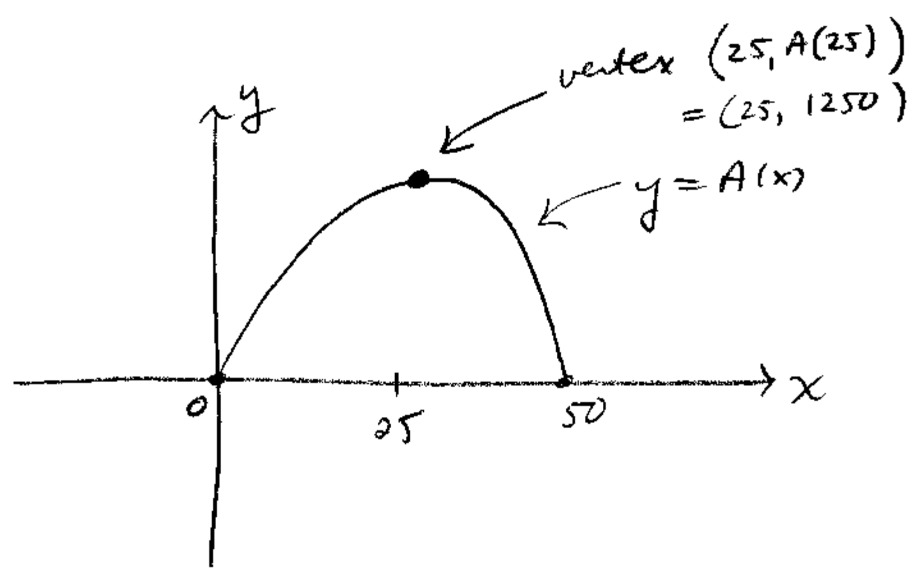
Question 12 [10 points]: A farmer wishes to build a fenced rectangular enclosure which is divided into three smaller rectangular enclosures of equal size as shown in the figure below. 200 metres of fencing is available for the project. Determine the maximum possible area enclosed.



$$\frac{200-4x}{2} = 100 - 2x$$

$$\therefore\ A(x) = x\,(100 - 2x)$$

$$= 2x\,(50 - x)$$



$\therefore$ Maximum area is $1250\ m^2$.

Question 13 [10 points]: Find all zeros of the polynomial function:

$$f(x) = x^4 - x^3 - 6x^2 + 4x + 8$$

p = 1, −1, 2, −2, 4, −4, 8, −8

s = 1, −1

∴ p/s = ~~1~~, ~~−1~~, 2, −2, 4, −4, 8, −8

x = 1:

1	1	−1	−6	4	8
		1	0	−6	−2
	1	0	−6	−2	6

×

x = −1:

−1	1	−1	−6	4	8
		−1	2	4	−8
	1	−2	−4	8	0

∴ $f(x) = (x+1)(x^3 - 2x^2 - 4x + 8)$

x = −1:

−1	1	−2	−4	8
		−1	3	1
	1	−3	−1	9

×

x = 2:

2	1	−2	−4	8
		2	0	−8
	1	0	−4	0

∴ $f(x) = (x+1)(x-2)(x^2-4)$

$= (x+1)(x-2)(x-2)(x+2)$

∴ zeros are $x = -1$, $x = 2$, $x = -2$,

Question 14 [10 points]: Solve and state your answer using interval notation:

$$\frac{1}{x+2} > \frac{3}{x+1}$$

$$\frac{1}{x+2} - \frac{3}{x+1} > 0$$

$$\frac{x+1-3(x+2)}{(x+2)(x+1)} > 0$$

$$\frac{-2x-5}{(x+2)(x+1)} > 0$$

$$\boxed{\frac{-(2x+5)}{(x+2)(x+1)} > 0}$$

$$-(2x+5) = 0 \Rightarrow x = \frac{-5}{2}$$

$$(x+2)(x+1) = 0 \Rightarrow x = -2, \quad x = -1$$

$-5/2 \qquad -2 \qquad -1$

$-5 \qquad -\frac{9}{4} \qquad -\frac{3}{2} \qquad 0$

$\frac{-(2x+5)}{(x+2)(x+1)}$ $\quad + \quad 0 \quad - \quad NA \quad + \quad NA \quad -$

$$\therefore \frac{-(2x+5)}{(x+2)(x+1)} > 0 \text{ on } \left(-\infty, -\tfrac{5}{2}\right) \cup (-2, -1)$$