

Question 1:

(a)[5 points] Rationalize the denominator in

$$\begin{aligned} & \frac{2-\sqrt{3}}{2+5\sqrt{3}} \cdot \frac{2-5\sqrt{3}}{2-5\sqrt{3}} \\ &= \frac{4-2\sqrt{3}-10\sqrt{3}+5\cdot 3}{4-25\cdot 3} \\ &= \frac{19-12\sqrt{3}}{-71} \\ &= \boxed{\frac{-19+12\sqrt{3}}{71}} \end{aligned}$$

(b)[5 points] Simplify. Express your answer so that only positive exponents appear.

$$\begin{aligned} & \frac{(xy)^{1/4}(x^2y^2)^{1/2}}{(x^2y)^{3/4}} \\ &= \frac{x^{1/4} y^{1/4} x^2 y^2}{x^{6/4} y^{3/4}} \\ &= x^{\frac{1}{4}+\frac{4}{4}-\frac{6}{4}} y^{\frac{1}{4}+\frac{4}{4}-\frac{3}{4}} \\ &= x^{-\frac{1}{4}} y^{\frac{2}{4}} \\ &= \boxed{\frac{y^{1/2}}{x^{1/4}}} \end{aligned}$$

Question 2:

(a)[4 points] Find the distance $d(P_1, P_2)$ between the points $P_1 = (-4, -3)$ and $P_2 = (6, 2)$.
 x_1, y_1 x_2, y_2

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{(6 - (-4))^2 + (2 - (-3))^2} \\&= \sqrt{100 + 25} \\&= \sqrt{125} = \boxed{5\sqrt{5}}\end{aligned}$$

(b)[3 points] Find the midpoint of the line segment joining the points $P_1 = (-4, -3)$ and $P_2 = (6, 2)$.

$$\begin{aligned}M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\&= \left(\frac{-4 + 6}{2}, \frac{-3 + 2}{2} \right) \\&= \boxed{\left(1, -\frac{1}{2} \right)}\end{aligned}$$

(c)[3 points] Find the slope of the line through the points $P_1 = (-4, -3)$ and $P_2 = (6, 2)$.

$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1} \\&= \frac{2 - (-3)}{6 - (-4)} \\&= \frac{5}{10} = \boxed{\frac{1}{2}}\end{aligned}$$

Question 3:

(a)[5 points] Find the x and y intercepts of $4x^2 + y^3 = 8$. x -intercepts: $y=0$:

$$4x^2 + 0 = 8$$

$$x^2 = 2$$

$$x = \sqrt{2}, -\sqrt{2}$$

 y -intercepts: $x=0$:

$$4 \cdot 0^2 + y^3 = 8$$

$$y^3 = 8$$

$$y = 2$$

(b)[5 points] Determine if the graph of $4x^2 + y^3 = 8$ is symmetric about the x -axis, the y -axis or the origin. x -axis:

$$4x^2 + y^3 = 8 \leftarrow$$

 $y \leftrightarrow -y:$

$$4x^2 + (-y)^3 = 8$$

$$4x^2 - y^3 = 8 \leftarrow$$

X

 y -axis:

$$4x^2 + y^3 = 8 \leftarrow$$

 $x \leftrightarrow -x:$

$$4(-x)^2 + y^3 = 8$$

$$4x^2 + y^3 = 8 \leftarrow$$

✓

origin:

$$4x^2 + y^3 = 8 \leftarrow$$

 $x \leftrightarrow -x,$ $y \leftrightarrow -y:$

$$4(-x)^2 + (-y)^3 = 8$$

$$4x^2 - y^3 = 8 \leftarrow$$

X

\therefore graph of $4x^2 + y^3 = 8$ is symmetric about the y -axis.

Question 4:

- (a) [5 points] Put the circle $x^2 + y^2 + 4x - 12y - 9 = 0$ into standard form and state the centre and radius.

$$\begin{aligned}x^2 + 4x + y^2 - 12y - 9 &= 0 \\(x+2)^2 - 4 + (y-6)^2 - 36 - 9 &= 0 \\(x+2)^2 + (y-6)^2 &= 49\end{aligned}$$

\therefore centre is $(-2, 6)$,
radius $r = 7$

- (b) [5 points] Find the equation of the line through $(1, -2)$ which is perpendicular to the line $y = -7x + 3$.

$$y = -7x + 3 \text{ has slope } m_1 = -7$$

$$\therefore \text{ line we want has slope } m = \frac{-1}{m_1} = \frac{1}{7}$$

\therefore Equation is

$$y - y_0 = m(x - x_0)$$

$$y - (-2) = \frac{1}{7}(x - 1)$$

$$\boxed{y + 2 = \frac{1}{7}(x - 1)}$$

$$\underline{\text{or}} \quad \boxed{y = \frac{1}{7}x - \frac{15}{7}}$$

Question 5:

(a)[5 points] Find the domain of the function $f(x) = \frac{3x}{\sqrt{1-3x}}$.

$$\text{must have } 1-3x > 0$$

$$1 > 3x$$

$$\frac{1}{3} > x$$

$$x < \frac{1}{3}$$

$$\therefore D(f) = (-\infty, \frac{1}{3})$$

(b)[5 points] Let $f(x) = \frac{1}{x}$. Find and simplify the difference quotient

$$\begin{aligned} & \frac{f(x+h) - f(x)}{h} \\ &= \frac{\left(\frac{1}{x+h} - \frac{1}{x}\right)}{h} \\ &= \frac{\left(\frac{x - (x+h)}{(x+h)x}\right)}{h} \\ &= \frac{\cancel{x} - x - h}{(x+h)x} \\ &= \frac{-h}{(x+h)x} \cdot \frac{1}{h} \\ &= \boxed{\frac{-1}{x(x+h)}} \end{aligned}$$