Question 1: [**40 points**] This question consists of 20 short answer problems each worth 2%. For each problem, clearly write your final answer in the box to the right. The solution to each problem is short, requiring no more space than that given.

(a) Express $\{x \mid x \geq -3\}$ using interval notation.

$\boxed{[-3, \infty)}$

(b) Simplify $\dfrac{(4n^{-1})^2(mn^3)^3}{2mn^5}$.

$$= \frac{16n^{-2}m^3n^9}{2mn^5}$$

$$= 2m^2n^2$$

$\boxed{8m^2n^2}$

(c) Simplify and give your answer in scientific notation: $\dfrac{(1.2 \times 10^{13})(-3.3 \times 10^{-4})}{7.2 \times 10^{15}}$.

$\boxed{-5.5 \times 10^{-7}}$

(d) Simplify $\frac{x^3 + 2x^2 - x - 2}{x + 2}$.

$$x^2 + 0x - 1$$
$$x+2 \overline{\smash{)}\, x^3 + 2x^2 - x - 2}$$
$$-(x^3 + 2x^2)$$
$$0x^2 - x$$
$$-(0x^2 + 0x)$$
$$-x - 2$$
$$-(-x - 2)$$
$$0$$

$$\boxed{x^2 - 1}$$

(e) Factor completely $4x^2 - y^4$.

$= (2x - y^2)(2x + y^2)$

$$\boxed{(2x - y^2)(2x + y^2).}$$

(f) Factor completely $2y^2 + 11y + 12$.

$= 2y^2 + 8y + 3y + 12$

$= 2y(y + 4) + 3(y + 4)$

$= (2y + 3)(y + 4)$

$$\boxed{(2y + 3)(y + 4).}$$

(g) Factor completely $3x^3 + 6x^2 - 27x - 54$.

$= 3x^2(x+2) - 27(x+2)$

$= (3x^2 - 27)(x+2)$

$= 3(x^2 - 9)(x+2)$

$= 3(x-3)(x+3)(x+2).$

$\boxed{3(x-3)(x+3)(x+2).}$

(h) Simplify: $\dfrac{\dfrac{x}{y} - \dfrac{y}{x}}{x+y}$.

$= \dfrac{x^2 - y^2}{(x+y)xy}$

$= \dfrac{(x-y)\cancel{(x+y)}}{\cancel{(x+y)}xy}$

$= \dfrac{x-y}{xy}$

$\boxed{\dfrac{x-y}{xy}}$

(i) Simplify $\dfrac{\sqrt[3]{3x^2}}{\sqrt[3]{24x^5}}$.

$= \sqrt[3]{\dfrac{1}{8x^3}}$

$= \dfrac{1}{2x}$

$\boxed{\dfrac{1}{2x}}$

(j) Rationalize the denominator: $\frac{p^2 - q^2}{\sqrt{p} - \sqrt{q}}$.

$$\frac{p^2-q^2}{\sqrt{p}-\sqrt{q}} \cdot \frac{\sqrt{p}+\sqrt{q}}{\sqrt{p}+\sqrt{q}}$$

$$= \frac{\cancel{(p-q)}(p+q)(\sqrt{p}+\sqrt{q})}{\cancel{(p-q)}}$$

$$\boxed{(p+q)(\sqrt{p}+\sqrt{q}).}$$

(k) If $(a, 9)$ is a point on the graph of $y = -2x^3 + 7$, what is the value of a?

$$9 = -2a^3 + 7$$
$$2 = -2a^3$$
$$a^3 = -1$$
$$a = -1$$

$$\boxed{a = -1}$$

(l) Let $f(x) = x^4 \overset{-}{\cancel{+}}2x^2 + 1$ and $g(x) = \sqrt{x+1}$. Find and simplify $(f \circ g)(x)$.

$$f(g(x)) = (\sqrt{x+1})^4 - 2(\sqrt{x+1})^2 + 1$$
$$= (x+1)^2 - 2(x+1) + 1$$
$$= x^2 + \cancel{2x} \cancel{+1} \cancel{-2x} \cancel{-2} \cancel{+1}$$
$$= x^2$$

$$\boxed{x^2}$$

(m) What is the domain of $f(x) = \dfrac{\sqrt{x+1}}{x+1}$?

$\therefore \quad x+1 > 0 \quad , \quad x \neq -1$

$\therefore \quad x > -1$

$\boxed{\therefore \ (-1, \infty)}$

(n) Find the equation of the line through $(-2, 3)$ and $(3, 7)$.

$m = \dfrac{7-3}{3-(-2)} = \dfrac{4}{5}$

$\therefore \ y - 7 = \dfrac{4}{5}(x-3)$

or $y = \dfrac{4}{5}x + \dfrac{23}{5}$

$\boxed{y - 7 = \dfrac{4}{5}(x-3).}$

(o) Solve $x - 3\sqrt{x} - 18 = 0$.

$(\sqrt{x} - 6)(\sqrt{x} + 3) = 0$

$\sqrt{x} = 6$, ~~$\sqrt{x} = -3$~~

$x = 36$

$\boxed{x = 36.}$

(p) Solve for x: $\frac{1}{x} - \frac{1}{x-1} = 4$

$$\boxed{x = \frac{1}{2}}$$

$$(x-1) - x = 4x(x-1)$$

$$-1 = 4x^2 - 4x$$

$$4x^2 - 4x + 1 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1)}}{2(4)} = \frac{4 \pm 0}{8} = \frac{1}{2}$$

(q) Solve the system of equations

① $3x + 4y = 6$
② $2x + 3y = 5$

$$① \Rightarrow y = \frac{6-3x}{4}$$

$$② \Rightarrow 2x + 3\left(\frac{6-3x}{4}\right) = 5$$

$$\boxed{\begin{aligned} x &= -2 \\ y &= 3 \end{aligned}}$$

$$2x + \frac{18}{4} - \frac{9x}{4} = 5$$

$$8x + 18 - 9x = 20$$

$$-x = +2$$

$$\boxed{x = -2}$$

$$\therefore \quad y = \frac{6-3(-2)}{4} = 3$$

(r) Suppose $f(x) = (x+5)^{1/3} - 4$. Find $f^{-1}(x)$.

$$\boxed{f^{-1}(x) = (x+4)^3 - 5}$$

$$y = (x+5)^{1/3} - 4$$

$$x = (y+5)^{1/3} - 4$$

$$x + 4 = (y+5)^{1/3}$$

$$(x+4)^3 = y + 5$$

$$y = (x+4)^3 - 5$$

(s) Solve: $x - 2 < \dfrac{5+3x}{2}$.

$2x - 4 < 5 + 3x$

$-x < 9$

$x > -9$

$x > -9$ or
$(-9, \infty)$.

(t) Suppose the point $(-1, 3)$ in on the graph of $y = f(x)$. Calculate $f(-1) - f^{-1}(3)$.

$f(-1) - f^{-1}(3)$

$= 3 - (-1)$

$= 4$

4.

Question 2: [10 points]
Find all solutions to the following equation. Clearly show all steps:

$$\sqrt{x+7} - \sqrt{x-2} = 3$$

$$(\sqrt{x+7})^2 = (3 + \sqrt{x-2})^2$$

$$\cancel{x} + \cancel{7} = \cancel{9} + 6\sqrt{x-2} + \cancel{x}\cancel{-2}$$

$$\sqrt{x-2} = 0$$

$$x = 2$$

Check:

$$\sqrt{2+7} - \sqrt{2-2} \quad \Big\{ \quad 3$$

$$\sqrt{9} - 0 \quad \Big\{ \quad 3$$

$$3 = 3 \quad \checkmark$$

Question 3: [10 points]
Find all solutions to the following system of equations:

$$\begin{aligned} x - y^2 &= -2 \quad ① \\ x - 2y &= 1 \quad ② \end{aligned}$$

Using ② : $x = 1 + 2y$

sub. into ① : $1 + 2y - y^2 = -2$

$$y^2 - 2y - 3 = 0$$

$$(y-3)(y+1) = 0$$

$$y = 3, \quad y = -1$$

using ② : $\underline{\underline{y = 3}}$; $x = 1 + 2y$

$= 1 + 2(3)$

$= 7$

$\therefore (7, 3)$

Check : ① : $7 - (3)^2 \; \{ \; -2$

$7 - 9 \quad -2$

$-2 = -2$ ✓

② : $7 - 2(3) \; \{ \; 1$

$1 = 1$ ✓

$\underline{\underline{y = -1}}$: $x = 1 + 2y$

$= 1 + 2(-1)$

$= -1$

$\therefore (-1, -1)$

Check : ① : $-1 - (-1)^2 \; \{ \; -2$

$-1 - 1 \quad -2$

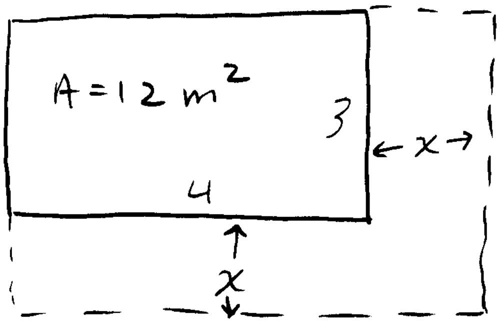
$-2 = -2$ ✓

② : $-1 - 2(-1) \; \{ \; 1$

$-1 + 2 \quad 1$

$1 = 1$ ✓

Question 4: [10 points]
A rectangular garden has a length of 4 m and width of 3 m. The garden is to be enlarged by extending its current length and width by equal amounts so that the area of the new garden is twice the area of the original. What are the dimensions of the new garden? Round your answer to 1 decimal place.



$$\therefore (3+x)(4+x) = 2(12)$$

$$12 + 7x + x^2 = 24$$

$$x^2 + 7x - 12 = 0$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-12)}}{2(1)}$$

$$= \frac{-7 \pm \sqrt{97}}{2}$$

$$= \frac{-7+\sqrt{97}}{2}, \quad \frac{-7-\sqrt{97}}{2}$$

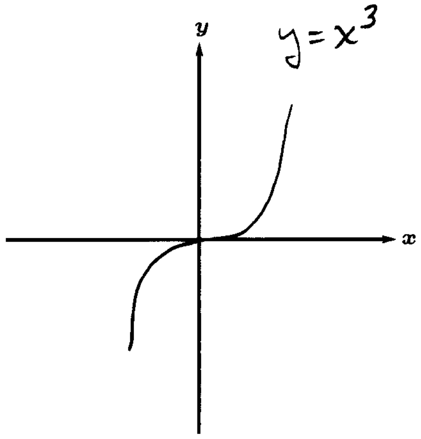
$$\doteq 1.4$$

$\therefore$ new length $= 4 + 1.4 = 5.4$ m

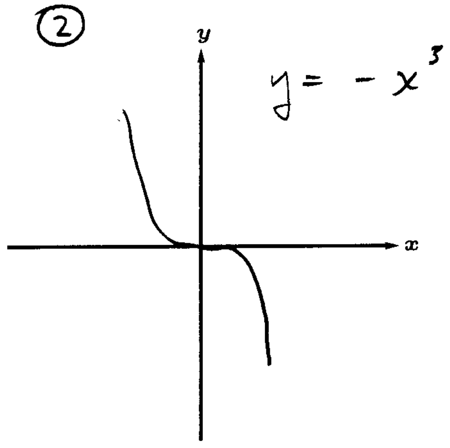
new width $= 3 + 1.4 = 4.4$ m

Question 5: [10 points]
(a)[**7 points**] Carefully graph the function $f(x) = -(x-1)^3 - 2$ below by starting with the graph of one of the basic functions and applying three transformations. On your final graph label at least one point as well as the y-intercept.

①



②



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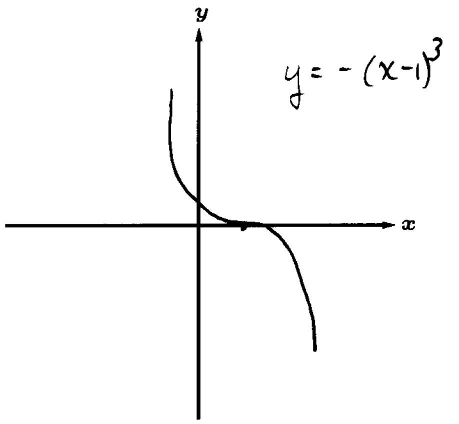
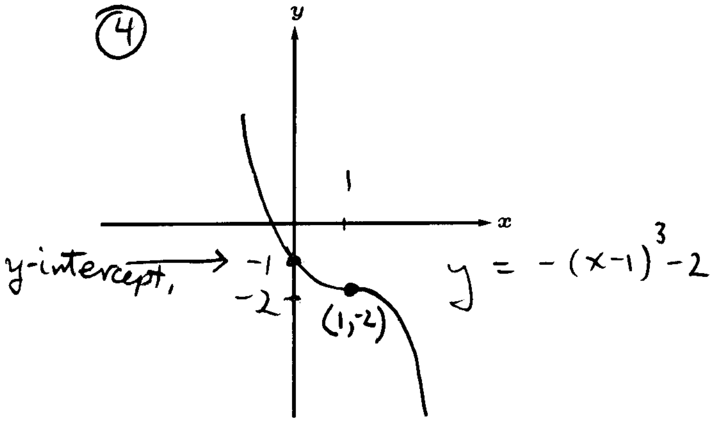
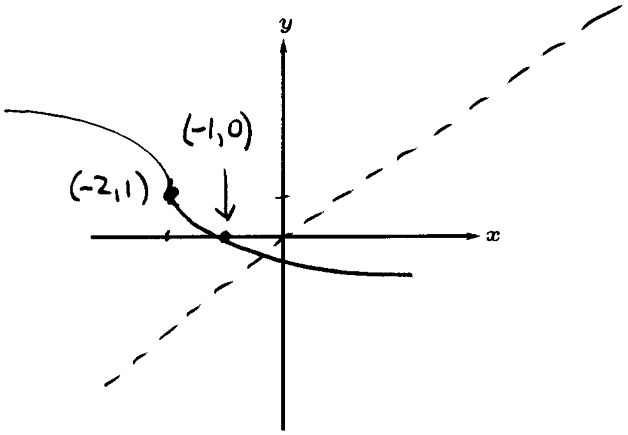


④



(b)[**3 points**] Using your answer in part (a), sketch the graph of $f^{-1}(x)$:



Question 6: [10 points]
Factor completely

$$f(x) = x^4 - 3x^3 - 3x^2 + 11x - 6$$

$f(1) = 0$, so $(x-1)$ is a factor:

$$\begin{array}{r}
x^3-2x^2-5x+6 \\
x-1\,\overline{)\,x^4-3x^3-3x^2+11x-6} \\
-(x^4-x^3)\downarrow \qquad\qquad \\
\hline
-2x^3-3x^2 \qquad\quad \\
-(-2x^3+2x^2)\downarrow \quad \\
\hline
-5x^2+11x \quad \\
-(-5x^2+5x)\downarrow \\
\hline
6x-6 \\
-(6x-6) \\
\hline
0
\end{array}$$

$$\therefore\ f(x) = (x-1)\underbrace{(x^3-2x^2-5x+6)}_{g(x).}$$

$g(1) = 0$, so $(x-1)$ is a factor:

$$\begin{array}{r}
x^2-x-6 \\
x-1\,\overline{)\,x^3-2x^2-5x+6} \\
-(x^3-x^2)\downarrow \qquad\quad \\
\hline
-x^2-5x \qquad \\
-(-x^2+x)\downarrow \\
\hline
-6x+6 \\
-(-6x+6) \\
\hline
0
\end{array}$$

$$\therefore\ f(x) = (x-1)^2(x^2-x-6) = \boxed{(x-1)^2(x+2)(x-3)}$$

Question 7: [10 points]
Solve

$$2x^2 + 3x - 9 < 0 \ .$$

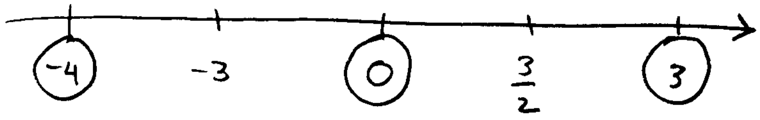
State your answer using interval notation, and clearly justify your conclusions.

$$2x^2 + 3x - 9 = 0$$

$$x = \frac{-3 \pm \sqrt{9 - 4(2)(-9)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{81}}{4}$$

$$= \frac{3}{2} \quad , \ -3$$



$2x^2+3x-9$: + − +

$\therefore \quad 2x^2 + 3x - 9 < 0 \quad$ on $\quad (-3, \frac{3}{2})$