Question 1: This question consists of 20 short answer problems each worth 2%. For each problem, clearly write your answer in the box to the right AS ONLY THAT ANSWER WILL BE GRADED. The solution to each problem is short, requiring no more space than that given. Although no part marks are awarded, show your work clearly in case it is needed to support your final answer.

(a) Find the equation of the line through the points $(3, -5)$ and $(5, 0)$.

$m = \frac{-5-0}{3-5} = \frac{-5}{-2} = \frac{5}{2}$

$\therefore \quad y - 0 = \frac{5}{2}(x-5)$

$y = \frac{5}{2}(x-5)$ or $y = \frac{5}{2}x - \frac{25}{2}$

$y = \frac{5}{2}x - \frac{25}{2}$

(b) Simplify $\frac{(x^2y^{-3})^5}{(x^{-2}y^5)^{-3}}$ and show your answer using only positive exponents.

$\frac{x^{10}y^{-15}}{x^6y^{-15}} = x^4$

x^4

(c) CDs are on sale for 15% off the regular price. A CD is purchased for \$13.99; what was the original price?

$13.99 = 0.85P$

$P = \frac{13.99}{0.85} = \$16.46$

OMIT

\$16.46

(d) Compute $\frac{(-6.23 \times 10^{11})(3.99 \times 10^{-7})}{2.70 \times 10^2}$ and give your answer in scientific notation rounded to three digits.

-9.21×10^2

OMIT

-9.21×10^2

(e) Solve for x: $-2x^2 - 6x + 20 = 0$.

$x^2 + 3x - 10 = 0$

$(x-2)(x+5) = 0$

$x = 2, \quad x = -5$

$x = -5, 2$

(f) Solve for x: $\left(\sqrt{2(x-1)}+1\right)^3 - 8 = 0$

$\left(\sqrt{2(x-1)}+1\right)^3 = 8$

$\sqrt{2(x-1)}+1 = 2$ OMIT

$2(x-1) = 1 \Rightarrow x = \frac{3}{2}$

$x - 1 = \frac{1}{2}$

$x = 3/2$

(g) Let $f(x) = (1+x)^{1/3}$. Simplify $f(x^3 - 1)$.

$f(x^3-1) = \left[1 + (x^3-1)\right]^{1/3}$

$= \left[x^3\right]^{1/3}$

$= x$

x

(h) A line through the points $(0, 0)$ and $(-3, a)$ is parallel to the line $y = -\frac{x}{3} + 137$. What is the value of a?

$\frac{a-0}{-3-0} = -\frac{1}{3}$

$\frac{-a}{3} = \frac{-1}{3}$

$a = 1$

$a = 1$

(i) Find the slope of the line defined by the equation $x + 2y + 1 = -3(x - y) - 7$.

$x + 2y + 1 = -3x + 3y - 7$

$4x - y + 8 = 0$

$y = 4x + 8$

$\therefore$ $m = 4$

$m = 4$

(j) What is the domain of the function $f(x) = \frac{x}{\sqrt{x+1}}$? Use interval notation for your answer.

$x + 1 > 0$

$x > -1$

$\therefore$ $(-1, \infty)$

$(-1, \infty)$.

(k) Find the point of intersection of the lines $y = -3x + \frac{1}{2}$ and $y = x - \frac{5}{2}$.

$-3x + \frac{1}{2} = x - \frac{5}{2}$

$3 = 4x$

$x = \frac{3}{4}$

$\therefore$ $y = \frac{3}{4} - \frac{5}{2} = \frac{3-10}{4} = \frac{-7}{4}$

$(\frac{3}{4}, -\frac{7}{4})$.

(l) The point $(1, 1)$ is on the graph of $f(x)$ and the point $(1, b)$ is on the graph of $-2f(x)+3$. What is the value of b?

$b = -2f(1) + 3$

$= (-2)(1) + 3$

$= 1$

$b = 1$

(m) Let $g(x) = \dfrac{1}{\sqrt{3x-7}} + 5$. Find $g^{-1}(x)$.

$y = \dfrac{1}{\sqrt{3x-7}} + 5$ $\quad$ $3x-7 = \left(\dfrac{1}{y-5}\right)^2$

$y - 5 = \dfrac{1}{\sqrt{3x-7}}$ $\quad$ $x = \dfrac{1}{3}\left[\left(\dfrac{1}{y-5}\right)^2 + 7\right]$

$\sqrt{3x-7} = \dfrac{1}{y-5}$ $\quad$ $x \leftrightarrow y$:

$\therefore\ y = \dfrac{1}{3}\left[\left(\dfrac{1}{x-5}\right)^2 + 7\right]$

$\boxed{y = \frac{1}{3}\left[\left(\frac{1}{x-5}\right)^2 + 7\right]}$

(n) Factor completely: $x^2 - 5x - 14$.

$(x+2)(x-7$

$\boxed{(x+2)(x-7)}$

(o) Rationalize the numerator and simplify: $\dfrac{\sqrt{2}-4}{14x}$.

$$\frac{\sqrt{2}-4}{14x} \cdot \frac{\sqrt{2}+4}{\sqrt{2}+4} = \frac{2-16}{14x(\sqrt{2}+4)}$$

$$= \frac{-14}{14x(\sqrt{2}+4)}$$

$\boxed{\dfrac{-1}{x(\sqrt{2}+4)}}$

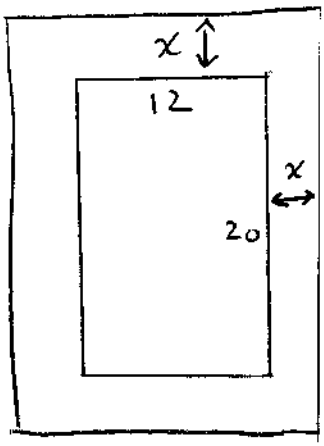
(p) Let $h(x) = \left(\sqrt{2x^2+1} - 1\right)^3 + \dfrac{1}{2}$ and $f(x) = x^3 + \dfrac{1}{2}$. If $h(x) = (f \circ g)(x)$, what is $g(x)$?

$g(x) = \sqrt{2x^2+1} - 1$

OMIT

$\boxed{g(x) = \sqrt{2x^2+1} - 1}$

Question 3: [10 points]
We wish to build a picture frame for a picture of size 12 cm by 20 cm. The frame is to be the same width on all sides, and we would like the total area of the frame and picture to be twice the area of the picture alone. How wide should the frame be? Round your answer to 1 decimal place.



$$(12+2x)(20+2x) = 2(12)(20)$$

$$240 + 64x + 4x^2 = 480$$

$$4x^2 + 64x - 240 = 0$$

$$x^2 + 16x - 60 = 0$$

$$x = \frac{-16 \pm \sqrt{256 - 4(1)(-60)}}{2}$$

$$= \frac{-16 \pm \sqrt{496}}{2}$$

$$= \frac{-16 + \sqrt{496}}{2} \quad \text{or} \quad \frac{-16 - \sqrt{496}}{2}$$

since $x > 0$

$$\cong \boxed{3.1 \text{ cm}}$$

Question 4: [10 points]
Factor completely:

$$x^3 - 2x^2 - 5x + 10$$

$\frac{P}{S} = 1, -1, 2, -2, 5, -5, 10, -10.$

1	1	-2	-5	10
		1	-1	-6
	1	-1	-6	4 ✗

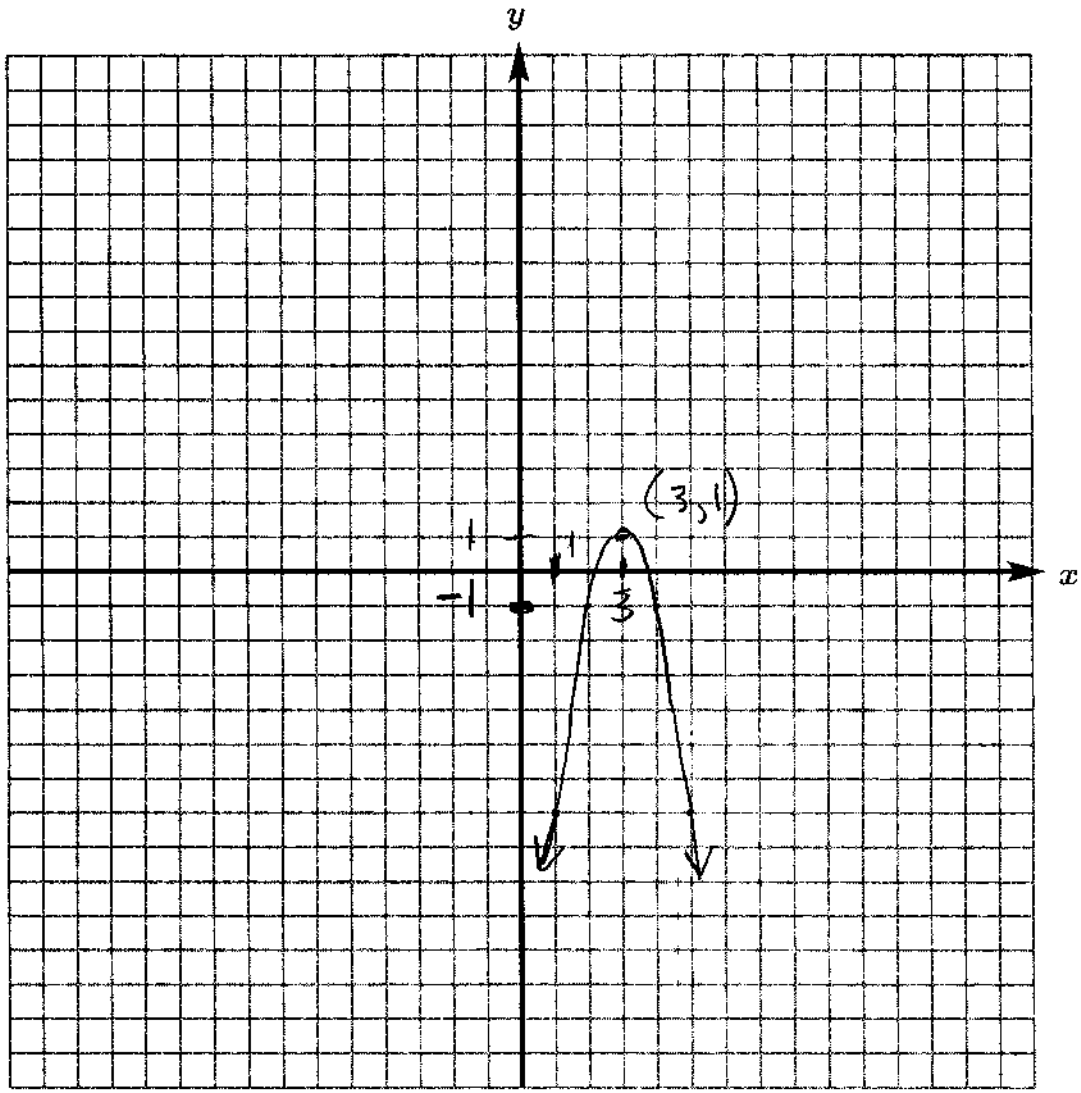
-1	1	-2	-5	10
		-1	3	2
	1	-3	-2	12 ✗

2	1	-2	-5	10
		2	0	-10
	1	0	-5	0 ✓

$\therefore x^3 - 2x^2 - 5x + 10 = (x-2)(x^2-5)$

$= (x-2)(x-\sqrt{5})(x+\sqrt{5})$

Question 5: [10 points]
(a)[7 points] Carefully graph the function $f(x) = -2(x-3)^2 + 1$ below. Label two points on your graph, and indicate the scale on the x and y axes.



(b)[3 points] What is the domain and range of the function $f(x)$ in part (a)?

Domain : $(-\infty, \infty)$

Range : $(-\infty, 1]$.

Question 7: [10 points]
Solve

$$6x^2 + x - 2 \geq 0 \, .$$

Give your answer using interval notation, and clearly justify your conclusions.

$6x^2 + x - 2 = 0$

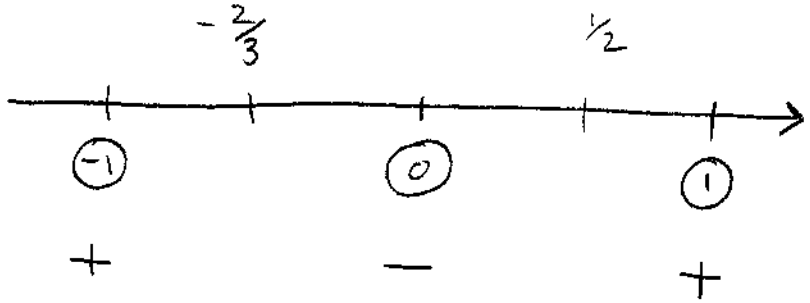
$6x^2 - 3x + 4x - 2 = 0$

$3x(2x-1) + 2(2x-1) = 0$

$(3x+2)(2x-1) = 0$

$3x + 2 = 0$, $2x - 1 = 0$

$x = \frac{-2}{3}$, $x = \frac{1}{2}$



$(3x+2)(2x-1)$: $+$ $-$ $+$

$\therefore \; 6x^2 + x - 2 \geq 0$ on $(-\infty, -\frac{2}{3}] \cup [\frac{1}{2}, \infty)$