Question 1: For this question consider the function $f(x) = -3x^2 + 5x$.

(a)[2 points] Is the point $(-1, 2)$ on the graph of f?

$f(-1) = -3(-1)^2 + 5(-1) = -8 \neq 2$,

so no.

(b)[2 points] What point on the graph of f has x-coordinate $x = -2$?

$f(-2) = -3(-2)^2 + 5(-2) = -22$.

$\therefore$ point is $(-2, -22)$.

(c)[3 points] Find all values of x for which $f(x) = -2$.

$-3x^2 + 5x = -2$

$-3x^2 + 5x + 2 = 0$

$3x^2 - 5x - 2 = 0$

$3x^2 - 6x + x - 2 = 0$

$3x(x-2) + (x-2) = 0$

$(3x+1)(x-2) = 0$

$x = \frac{-1}{3}$, $x = 2$

(d)[3 points] Find the x-intercepts of the graph of f.

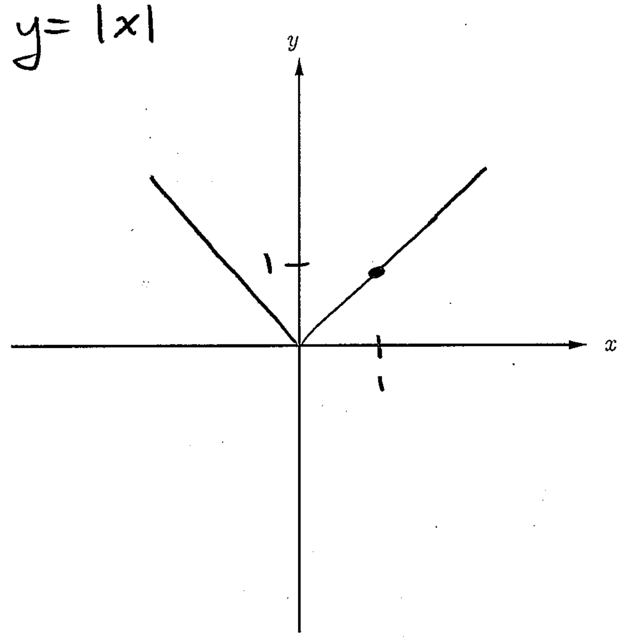
$-3x^2 + 5x = 0$

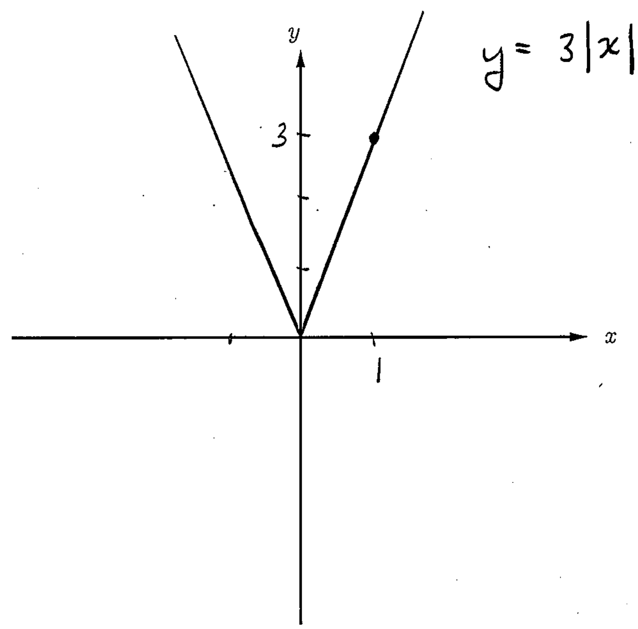
$x(-3x+5) = 0$

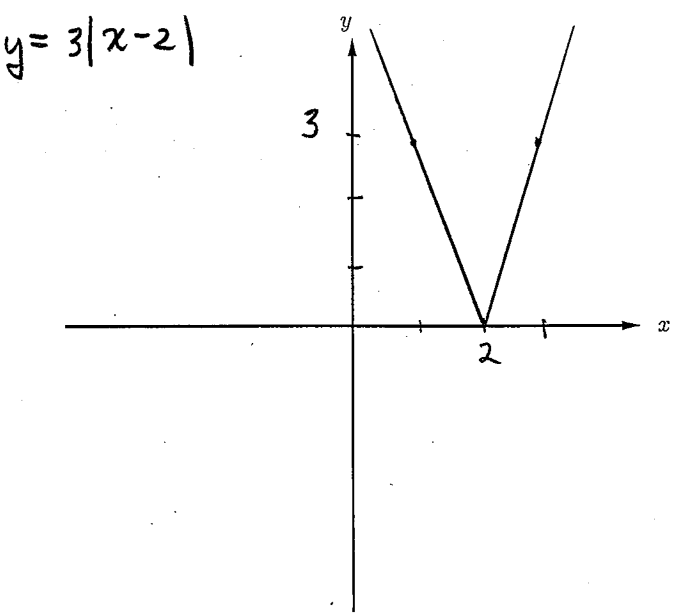
$x = 0$, $-3x + 5 = 0$

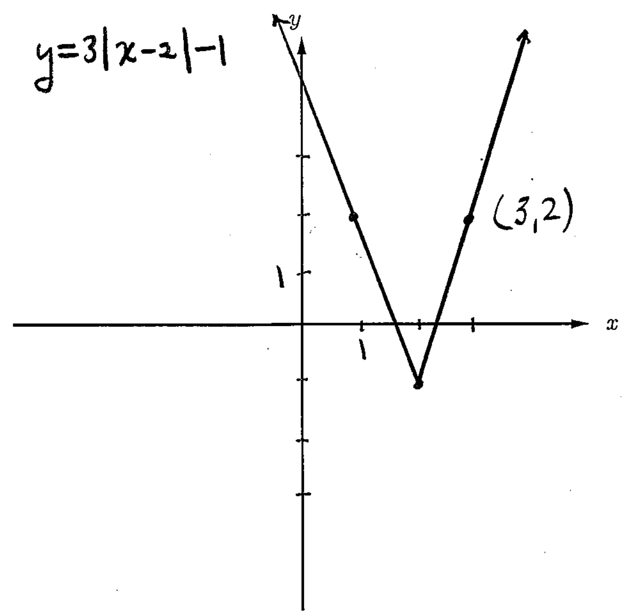
$x = 0$, $x = \frac{5}{3}$

Question 2 [10 points]: Neatly sketch the graph of the function $f(x) = 3|x-2| - 1$ below by starting with a basic function and applying three transformations. Your final answer should appear in the last graph below. In your final graph indicate the scale on the x and y axes and label at least one point.









Question 3: The supply function for a particular good is $S(p) = -100 + 20p$, while the demand function for the same good is $D(p) = 600 - 15p$. Here p is the price of the good in dollars.

(a) **[5 points]** Find the equilibrium price and quantity.

$$-100 + 20p = 600 - 15p$$
$$35p = 700$$
$$p = \frac{700}{35} = \$20$$
$$\therefore q = -100 + 20(20)$$
$$= 300$$

$\therefore$ Equilibrium price is \$20, equilibrium quantity is 300 units.

(b) **[5 points]** Determine the prices for which quantity demanded is greater than quantity supplied.

Solve $D(p) > S(p)$:

$$600 - 15p > -100 + 20p$$
$$700 > 35p$$
$$20 > p$$

$\therefore$ if $p < \$20$ quantity demanded is greater than quantity supplied,

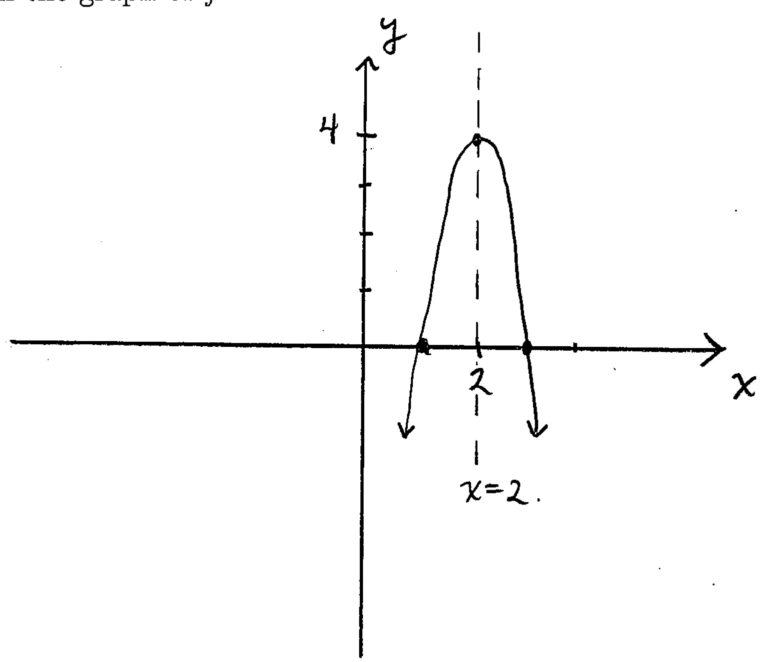
Question 4: For this question consider the quadratic function $f(x) = -4x^2 + 16x - 12$

(a)[**5 points**] Find the vertex and axis of symmetry of the graph of f.

$$\begin{aligned} f(x) &= -4x^2 + 16x - 12 \\ &= -4\left[x^2 - 4x + 3\right] \\ &= -4\left[(x-2)^2 - 4 + 3\right] \\ &= -4(x-2)^2 + 4 \end{aligned}$$

$\therefore$ vertex is $(2,4)$, axis of symmetry is $x=2$.

(b)[**5 points**] Sketch the graph of f.



Question 5: The price p and quantity sold x of a certain product obey the demand equation

$$p = -\frac{1}{5}x + 10, \qquad \text{where } 0 \le x \le 50 .$$

(a)[3 points] Find an expression (a formula) for $R(x)$, the revenue as a function of x.

$$R(x) = p \cdot x$$
$$= \left(-\frac{1}{5}x + 10\right) x$$
$$= -\frac{1}{5}x^2 + 10x$$

(b)[5 points] Determine the maximum revenue.

$$R(x) = -\frac{1}{5}x^2 + 10x$$
$$= -\frac{1}{5}\left[x^2 - 50x\right]$$
$$= -\frac{1}{5}\left[(x-25)^2 - 625\right]$$
$$= -\frac{1}{5}(x-25)^2 + 125$$

$\therefore$ maximum revenue is \$125.

(c)[2 points] What price should be charged to maximize revenue?

$p = -\frac{1}{5}(25) + 10 = \$5$ yields max. revenue.