Question 1:

(a)[5 points] Rationalize the denominator in

$$\frac{2-\sqrt{3}}{2+5\sqrt{3}} \cdot \frac{2-5\sqrt{3}}{2-5\sqrt{3}}$$

$$= \frac{4-2\sqrt{3}-10\sqrt{3}+5\cdot 3}{4-25\cdot 3}$$

$$= \frac{19-12\sqrt{3}}{-71}$$

$$= \boxed{\frac{-19+12\sqrt{3}}{71}}$$

(b)[5 points] Simplify. Express your answer so that only positive exponents appear.

$$\frac{(xy)^{1/4}(x^2y^2)^{1/2}}{(x^2y)^{3/4}}$$

$$= \frac{x^{\frac{1}{4}} y^{\frac{1}{4}} x y}{x^{\frac{6}{4}} y^{\frac{3}{4}}}$$

$$= x^{\frac{1}{4}+\frac{4}{4}-\frac{6}{4}} y^{\frac{1}{4}+\frac{4}{4}-\frac{3}{4}}$$

$$= x^{-\frac{1}{4}} y^{\frac{2}{4}}$$

$$= \boxed{\frac{y^{1/2}}{x^{1/4}}}$$

Question 2:

(a)[**4 points**] Find the distance $d(P_1, P_2)$ between the points $P_1 = (-4, -3)$ and $P_2 = (6, 2)$.

x_1, y_1 $\quad$ x_2, y_2

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(6-(-4))^2 + (2-(-3))^2}$$
$$= \sqrt{100 + 25}$$
$$= \sqrt{125} = \boxed{5\sqrt{5}}$$

(b)[**3 points**] Find the midpoint of the line segment joining the points $P_1 = (-4, -3)$ and $P_2 = (6, 2)$.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{-4+6}{2}, \frac{-3+2}{2}\right)$$
$$= \boxed{\left(1, \frac{-1}{2}\right)}$$

(c)[**3 points**] Find the slope of the line through the points $P_1 = (-4, -3)$ and $P_2 = (6, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2-(-3)}{6-(-4)}$$
$$= \frac{5}{10} = \boxed{\frac{1}{2}}$$

Question 3:

(a)[5 points] Find the x and y intercepts of $4x^2 + y^3 = 8$.

x-intercepts: $y=0$:

$4x^2 + 0 = 8$

$x^2 = 2$

$\boxed{x = \sqrt{2}, -\sqrt{2}}$

y-intercepts: $x=0$:

$4 \cdot 0^2 + y^3 = 8$

$y^3 = 8$

$\boxed{y = 2}$

(b)[5 points] Determine if the graph of $4x^2 + y^3 = 8$ is symmetric about the x-axis, the y-axis or the origin.

x-axis:

$4x^2 + y^3 = 8$

$y \leftrightarrow -y$:

$4x^2 + (-y)^3 = 8$

$4x^2 - y^3 = 8$

X

y-axis:

$4x^2 + y^3 = 8$

$x \leftrightarrow -x$:

$4(-x)^2 + y^2 = 8$

$4x^2 + y^2 = 8$

✓

origin:

$4x^2 + y^3 = 8$

$x \leftrightarrow -x$,
$y \leftrightarrow -y$:

$4(-x)^2 + (-y)^3 = 8$

$4x^2 - y^3 = 8$

X

$\therefore$ graph of $4x^2 + y^3 = 8$ is symmetric about the y-axis.

Question 4:

(a)[5 points] Put the circle $x^2 + y^2 + 4x - 12y - 9 = 0$ into standard form and state the centre and radius.

$$x^2+4x+y^2-12y-9=0$$

$$(x+2)^2-4+(y-6)^2-36-9=0$$

$$(x+2)^2+(y-6)^2=49$$

$\therefore$ centre is $(-2,6)$,

radius $r=7$

(b)[5 points] Find the equation of the line through $(1,-2)$ which is perpendicular to the line $y = -7x + 3$.

$y=-7x+3$ has slope $m_1=-7$

$\therefore$ line we want has slope $m=\frac{-1}{m_1}=\frac{1}{7}$

$\therefore$ Equation is

$$y-y_0=m(x-x_0)$$

$$y-(-2)=\frac{1}{7}(x-1)$$

$$\boxed{y+2=\frac{1}{7}(x-1)}$$

or $\boxed{y=\frac{1}{7}x-\frac{15}{7}}$

Question 5:

(a)[5 points] Find the domain of the function $f(x) = \dfrac{3x}{\sqrt{1-3x}}$.

must have $1-3x > 0$

$1 > 3x$

$\frac{1}{3} > x$

$x < \frac{1}{3}$

$\boxed{\therefore\ D(f) = (-\infty, \tfrac{1}{3}).}$

(b)[5 points] Let $f(x) = \dfrac{1}{x}$. Find and simplify the difference quotient

$$\frac{f(x+h)-f(x)}{h}.$$

$$= \frac{\left(\frac{1}{x+h} - \frac{1}{x}\right)}{h}$$

$$= \frac{\left(\frac{x-(x+h)}{(x+h)x}\right)}{h}$$

$$= \frac{\frac{\cancel{x}-\cancel{x}-h}{(x+h)x}}{h}$$

$$= \frac{-\cancel{h}}{(x+h)\,x} \cdot \frac{1}{\cancel{h}}$$

$$= \boxed{\frac{-1}{x(x+h)}}$$