Question 1:

(a)[5 points] Simplify. Express your answer so that all exponents are positive.

$$\frac{4x^{-2}(yz)^{-1}}{2^3x^4y}$$

$$= \frac{4x^{-2}y^{-1}z^{-1}}{8x^4y}$$

$$= \boxed{\frac{1}{2x^6y^2z}}$$

(b)[5 points] Solve and state your answer using interval notation:

$$3 - 4(1 - x) \leq 4$$

$$3-4+4x \leq 4$$

$$4x-1 \leq 4$$

$$4x \leq 5$$

$$x \leq \frac{5}{4}$$

$$\boxed{\therefore \left(-\infty, \frac{5}{4}\right]}$$

Question 2:

(a)[**5 points**] Find the quotient and remainder when $5x^4 - x^3 + x - 2$ is divided by $x^2 + 2$. In your answer clearly state which is the quotient and which is the remainder.

$$
\begin{array}{r}
5x^2 - x - 10 \\
x^2 + 0x + 2 \,\big)\, \overline{5x^4 - x^3 + 0x^2 + x - 2} \\
-(5x^4 + 0x^3 + 10x^2) \\
\hline
-x^3 - 10x^2 + x - 2 \\
-(-x^3 + 0x^2 - 2x) \\
\hline
-10x^2 + 3x - 2 \\
-(-10x^2 + 0x - 20) \\
\hline
3x + 18
\end{array}
$$

$\therefore$ quotient is $5x^2 - x - 10$

remainder is $3x + 18$

(b)[**5 points**] Determine if $x+3$ is a factor of $2x^6 - 18x^4 + x^2 - 9$. Clearly state and give a reason for your conclusion.

$$
\begin{array}{r|rrrrrrr}
\underline{-3} & 2 & 0 & -18 & 0 & 1 & 0 & -9 \\
 & & -6 & 18 & 0 & 0 & -3 & 9 \\
\hline
 & 2 & -6 & 0 & 0 & 1 & -3 & \boxed{0}
\end{array}
$$

Since remainder is zero, $x+3$ <u>is</u> a factor.

Question 3:

(a)[5 points] Factor completely:

$$3y^3 - 18y^2 - 48y$$

$$= 3y(y^2 - 6y - 16)$$

$$= \boxed{3y(y-8)(y+2)}$$

(b)[5 points] Factor completely:

$$x^8 - x^5$$

$$= x^5(x^3 - 1)$$

$$= \boxed{x^5(x-1)(x^2+x+1)}$$

Question 4:

(a)[5 points] Solve for t:

$$8t^2 - 2t - 3 = 0$$

$$8t^2 + 4t - 6t - 3 = 0$$

$$4t(2t+1) - 3(2t+1) = 0$$

$$(2t+1)(4t-3) = 0$$

$$\therefore \quad 2t+1 = 0 \qquad\qquad 4t-3=0$$

$$2t = -1 \qquad\qquad 4t = 3$$

$$t = \frac{-1}{2}, \qquad\qquad t = \frac{3}{4}$$

$$\boxed{\therefore \text{ Solutions are } t = \frac{-1}{2}, \ t = \frac{3}{4}}$$

(b)[5 points] Simplify:

$$\frac{3x}{x-1} - \frac{x-4}{x^2-2x+1}$$

$$= \frac{3x}{x-1} - \frac{x-4}{(x-1)(x-1)}$$

$$= \frac{3x(x-1) - (x-4)}{(x-1)(x-1)}$$

$$= \frac{3x^2 - 3x - x + 4}{(x-1)^2}$$

$$= \boxed{\frac{3x^2 - 4x + 4}{(x-1)^2}}$$

Question 5:

(a)[5 points] Solve for x:

$$4x^2 = 1 - 2x$$

$$4x^2 + 2x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{2^2 - 4(4)(-1)}}{2(4)}$$

$$= \frac{-2 \pm \sqrt{20}}{8}$$

$$= \frac{-2 \pm 2\sqrt{5}}{8} = \boxed{\frac{-1 \pm \sqrt{5}}{4}}$$

(b)[5 points] Solve for w:

$$\frac{5}{w+4} = 4 + \frac{3}{w-2}$$

$$w \neq -4, \quad w \neq 2$$

$$5(w-2) = 4(w+4)(w-2) + 3(w+4)$$

$$5w - 10 = 4(w^2 + 2w - 8) + 3w + 12$$

$$0 = 4w^2 + 8w - 32 + 3w + 12 - 5w + 10$$

$$4w^2 + 6w - 10 = 0$$

$$2(2w^2 + 3w - 5) = 0$$

$$2w^2 - 2w + 5w - 5 = 0$$

$$2w(w-1) + 5(w-1) = 0$$

$$(w-1)(2w+5) = 0$$

$$\therefore w - 1 = 0, \quad 2w + 5 = 0$$

$$\boxed{w = 1, \quad w = \frac{-5}{2}}$$