Question 1:

(a)[3 points] Find the slope of the line through the points $(5, -7)$ and $(-3, -1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-1-(-7)}{-3-5}$$

$$= \frac{6}{-8}$$

$$= \boxed{-\frac{3}{4}}$$

(b)[3 points] Simplify and express your answer so that all exponents are positive:

$$\frac{(x^{-1}y)^{1/3}}{(x^{2/3}y^{-2/3})^2}$$

$$= \frac{x^{-1/3}\,y^{1/3}}{x^{4/3}\,y^{-4/3}}$$

$$= \boxed{\frac{y^{5/3}}{x^{5/3}}}$$

(c)[4 points] Find the remainder when $-3x^4 + x^2 + 1$ is divided by $x^3 + x + 1$.

$$\begin{array}{r} -3x \\ x^3+0x^2+x+1 \,\big)\, -3x^4+0x^3+x^2+0x+1 \\ -(-3x^4+0x^3-3x^2-3x) \downarrow \\ \hline 4x^2+3x+1 \end{array}$$

$$\boxed{\therefore\ r(x) = 4x^2+3x+1}$$

Question 2:

(a)[3 points] Factor completely:

$$x^2 + 4x - 21$$

$$= \boxed{(x+7)(x-3)}$$

(b)[3 points] Factor completely:

$$10x^2 - 13x + 3$$

$$= 10x^2 - 10x - 3x + 3$$

$$= 10x(x-1) - 3(x-1)$$

$$= \boxed{(10x-3)(x-1)}$$

(c)[4 points] Factor completely:

$$x^3 + 6x^2 - 5x - 30$$

$$= x^2(x+6) - 5(x+6)$$

$$= (x^2-5)(x+6)$$

$$= \boxed{(x-\sqrt{5})(x+\sqrt{5})(x+6)}$$

Question 3:

(a)[3 points] Solve:

$$\frac{2x}{x-5} = 1 - 3x$$

$2x = (1-3x)(x-5)$, $x \neq 5$

$2x = x - 3x^2 - 5 + 15x$

$3x^2 - 14x + 5 = 0$

$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(3)(5)}}{2(3)}$

$= \frac{14 \pm \sqrt{196 - 60}}{6}$

$= \frac{14 \pm \sqrt{136}}{6} = \frac{14 \pm 2\sqrt{34}}{6} = \boxed{\frac{7 \pm \sqrt{34}}{3}}$

(b)[3 points] Solve:

$$1 + \frac{2-x}{3} > 5$$

$\frac{2-x}{3} > 4$

$2 - x > 12$

$-x > 10$

$x < -10$

$\boxed{\therefore (-\infty, -10)}$

(c)[4 points] Solve:

$$\left|\frac{4x+2}{7}\right| > 2$$

$\frac{4x+2}{7} > 2$ or $\frac{4x+2}{7} < -2$

$4x + 2 > 14$; $4x + 2 < -14$

$4x > 12$; $4x < -16$

$x > 3$; $x < -4$

$\therefore \boxed{(-\infty, -4) \cup (3, \infty)}$

Question 4:

(a)[3 points] Simplify:

$$\frac{\frac{x}{x+2}}{\frac{x-1}{x^2+x-2}}$$

$$= \frac{x}{x+2} \cdot \frac{x^2+x-2}{x-1}$$

$$= \frac{x}{\cancel{x+2}} \cdot \frac{\cancel{(x+2)}\cancel{(x-1)}}{\cancel{(x-1)}}$$

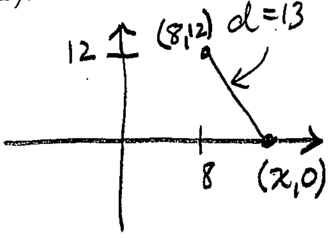
$$= \boxed{x}$$

(b)[3 points] Rationalize the denominator:

$$\frac{\sqrt{x}}{x+\sqrt{h}} \cdot \frac{x-\sqrt{h}}{x-\sqrt{h}}$$

$$= \frac{\sqrt{x}\,(x-\sqrt{h})}{x^2-h}$$

(c)[4 points] Find all points on the x-axis a distance 13 from the point $(8, 12)$.



$$13 = \sqrt{(x-8)^2 + (0-12)^2}$$

$$169 = (x-8)^2 + 144$$

$$25 = (x-8)^2$$

$\therefore\ x-8=5 \quad , \quad x-8=-5$

$x = 13 \quad , \quad x = 3$

$\therefore$ points are $(13,0)$ & $(3,0)$

Question 5:

(a)[3 points] Find the equation of the line through $(-4, 3)$ which is parallel to the line $y = \frac{2}{3}x - 121$.

Line has slope $m = \frac{2}{3}$.

$\therefore \boxed{y - 3 = \frac{2}{3}(x + 4)}$

or $\boxed{y = \frac{2}{3}x + \frac{17}{3}}$

(b)[3 points] Find the x-intercepts of $y^2 = x^3 - 4x$.

$$0 = x^3 - 4x$$

$$0 = x(x^2 - 4)$$

$$0 = x(x-2)(x+2)$$

$\therefore \boxed{x = 0, 2, -2.}$

(c)[4 points] Put the circle $x^2 + 6x + y^2 - 5y = \frac{3}{4}$ into standard form and state the radius.

$$(x+3)^2 - 9 + \left(y - \frac{5}{2}\right)^2 - \frac{25}{4} = \frac{3}{4}$$

$$(x+3)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{3}{4} + \frac{25}{4} + 9$$

$$(x+3)^2 + \left(y - \frac{5}{2}\right)^2 = 16$$

$\therefore$ radius is $r = 4$

Question 6:

(a)[3 points] Let $f(x) = x^2 + 1$. Compute and simplify

$$\frac{f(x+h) - f(x)}{h}$$

$$= \frac{(x+h)^2 + 1 - x^2 - 1}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 \cancel{+1} - \cancel{x^2} \cancel{-1}}{h}$$

$$= 2x + h$$

(b)[3 points] Determine the domain of $f(x) = \frac{\sqrt{x+1}}{x^2 - 1}$.

Must have $x+1 > 0$, and $x^2 - 1 \neq 0$.

$x^2 - 1 = 0 \Rightarrow (x-1)(x+1) = 0 \Rightarrow x = 1, x = -1.$

$\therefore$ must have $x > -1$, $x \neq 1$, $x \neq -1$.

$\therefore$ domain is $(-1, 1) \cup (1, \infty)$.

(c)[4 points] Determine the vertex of $f(x) = 2x^2 + 8x - 1$.

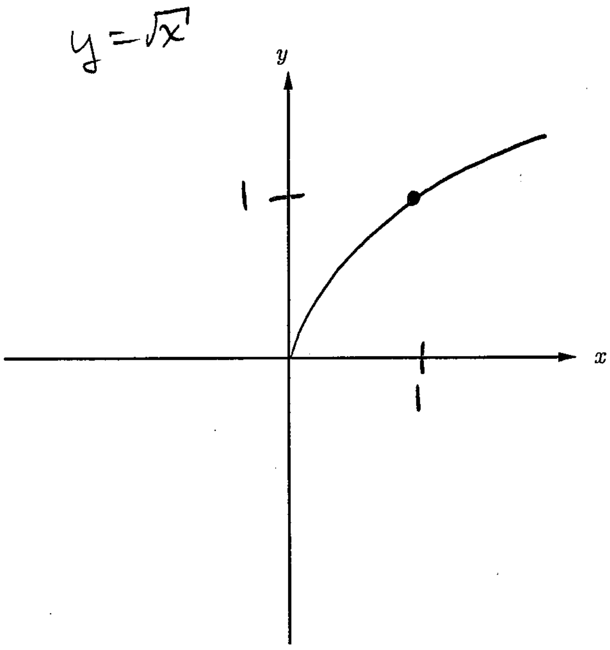
$$f(x) = 2\left[x^2 + 4x - \tfrac{1}{2}\right]$$

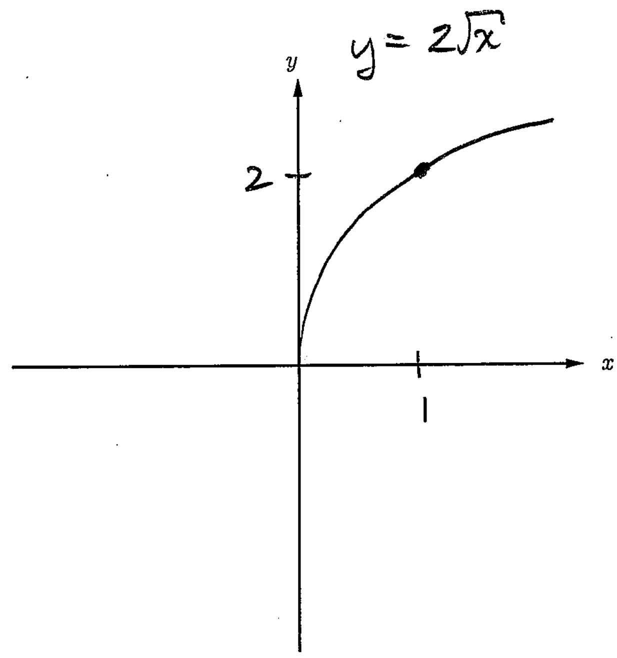
$$= 2\left[(x+2)^2 - 4 - \tfrac{1}{2}\right]$$

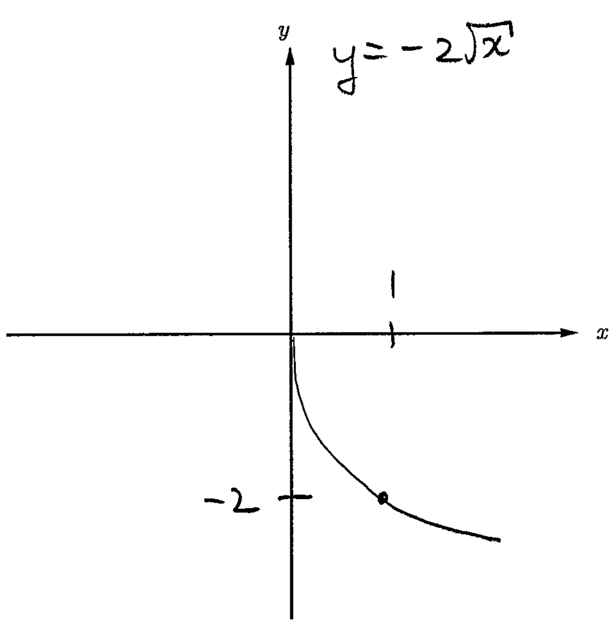
$$= 2(x+2)^2 - 9$$

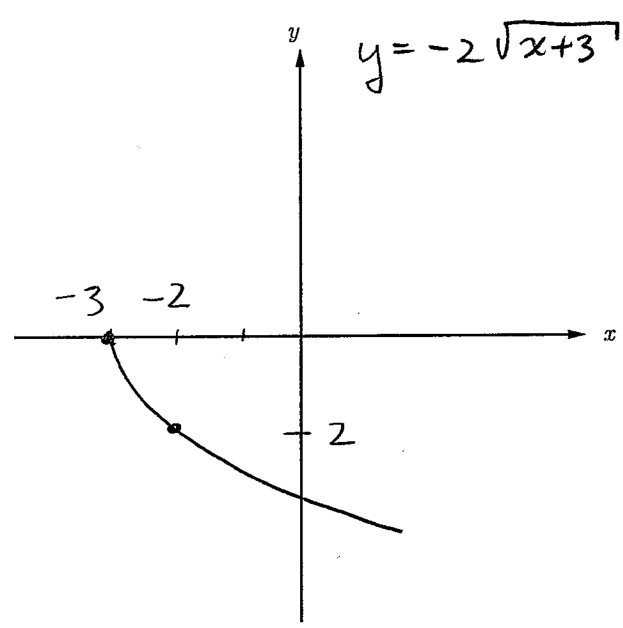
$\therefore$ vertex is $(-2, -9)$.

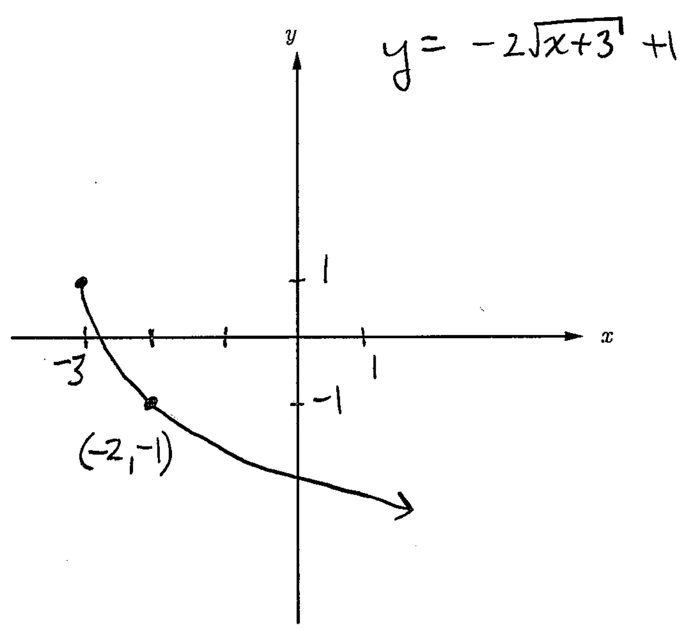
Question 7 [10 points]: Neatly sketch the graph of the function $f(x) = -2\sqrt{x+3}+1$ below by starting with a basic function and applying four transformations. Your final answer should appear in the last graph below. In your final graph indicate the scale on the x and y axes and label at least one point.











Question 8: The supply and demand functions for a particular product are given by

$$S(p) = -50 + 10p$$
$$D(p) = 220 - 8p$$

(a) [**5 points**] Determine the equilibrium price and quantity.

$$-50 + 10p = 220 - 8p$$
$$18p = 270$$
$$p = \frac{270}{18} = \$15$$

$$\therefore q = S(15) = -50 + 10(15)$$
$$= 100 \text{ units.}$$

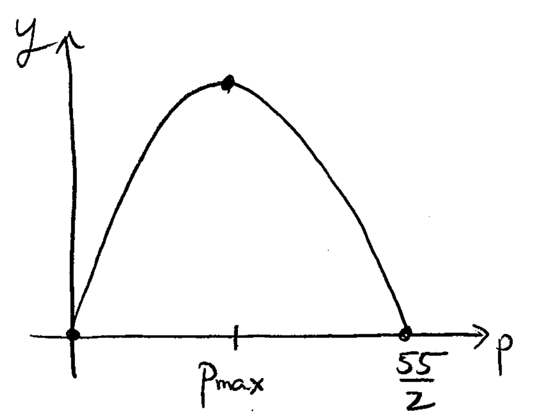
$$\therefore p = \$15, \quad q = 100 \text{ units.}$$

(b) [**5 points**] Since revenue is the product of price and quantity, the demand function above can be used to express revenue as a function of price: $R(p) = 220p - 8p^2$. Determine the price p which maximizes revenue.

$$R(p) = 220p - 8p^2$$
$$= p(220 - 8p)$$

$$p(220 - 8p) = 0$$
$$p = 0, \quad 220 - 8p = 0$$
$$220 = 8p$$
$$p = \frac{220}{8} = \frac{55}{2}$$

$\therefore$ $y = R(p)$ has graph



$\therefore$ revenue is maximized at $p = \frac{1}{2}\left(\frac{55}{2}\right) = \frac{55}{4} = \boxed{\$13.75}$

Question 9 [10 points]: Find all zeros of the polynomial function

$$f(x) = x^4 - 3x^3 - 3x^2 + 11x - 6 \, .$$

Neatly show all steps in your solution and state a clear conclusion.

$\frac{P}{S} = 1, -1, 2, -2, 3, -3, 6, -6$

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & -3 & 11 & -6 \\ & & 1 & -2 & -5 & 6 \\ \hline & 1 & -2 & -5 & 6 & \boxed{0} \end{array}$$

$\therefore f(x) = (x-1)(x^3 - 2x^2 - 5x + 6)$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 6 \\ & & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array}$$

$\therefore f(x) = (x-1)(x-1)(x^2 - x - 6)$

$= (x-1)(x-1)(x-3)(x+2)$

$\therefore f(x) = 0$ at $x = 1$, $x = 3$, $x = -2$.

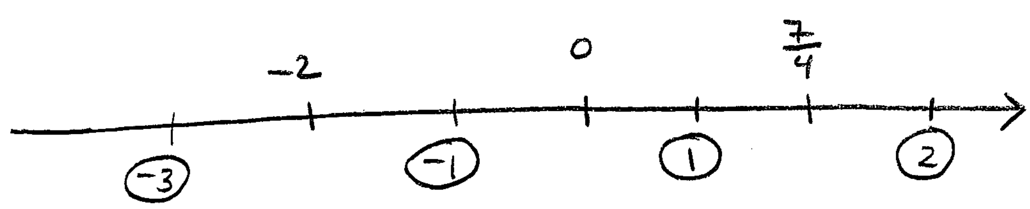
Question 10 [10 points]: Solve and state your answer using interval notation:

$$\frac{(4x-7)(x+2)}{x} \leq 0$$

Neatly show all steps in your solution and state a clear conclusion.

$4x - 7 = 0$, $x + 2 = 0$, $x = 0$

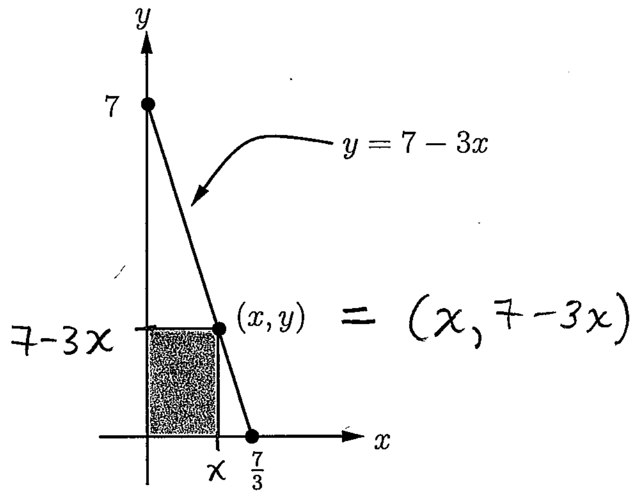
$x = \frac{7}{4}$, $x = -2$, $x = 0$



$\frac{(4x-7)(x+2)}{x}$: $-$ 0 $+$ NA $-$ 0 $+$

$\therefore \frac{(4x-7)(x+2)}{x} \leq 0$ on $(-\infty, -2] \cup (0, \frac{7}{4}]$

Question 11 [**10 points**]: A rectangle is inscribed in the first quadrant under the line $y = 7 - 3x$ as shown below. Determine the maximum possible area of such a rectangle.



Let $A(x)$ = area of rectangle.

$$A(x) = x(7-3x), \quad 0 \le x \le \frac{7}{3}.$$

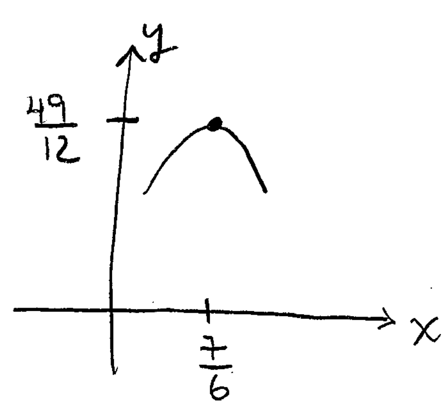
$$= 7x - 3x^2$$

$$= -3\left[x^2 - \frac{7}{3}x\right]$$

$$= -3\left[\left(x - \frac{7}{6}\right)^2 - \frac{49}{36}\right]$$

$$= -3\left(x - \frac{7}{6}\right)^2 + \frac{49}{12}$$

$\therefore$ $y = A(x)$ has graph



$\therefore$ maximum of $A(x)$ is $\frac{49}{12}$ square units.