Question 1:

(a)[3 points] Find the average value of $f(x) = 4x^3 + \frac{2}{x}$ on the interval $[1, e]$.

$$f_{ave} = \frac{1}{e-1}\int_1^e 4x^3 + \frac{2}{x}\,dx$$

$$= \frac{1}{e-1}\left[x^4 + 2\ln x\right]_1^e$$

$$= \frac{1}{e-1}\left[(e^4+2)-(1+0)\right] = \boxed{\frac{e^4+1}{e-1}}$$

(b)[3 points] Compute $F'(1)$ if $F(x) = \int_0^{x^2} e^{\sqrt{t}}\,dt$

$$F'(x) = e^{\sqrt{x^2}} \cdot 2x$$

$$F'(1) = e^{\sqrt{1^2}} \cdot 2$$

$$= \boxed{2e}$$

(c)[4 points] Evaluate $\int_{-\pi/2}^{\pi/2} \sqrt{\sin x + 1}\cos x\,dx$.

$u = \sin x + 1$ $\qquad x = -\frac{\pi}{2} \Rightarrow u = 0$

$du = \cos x\,dx$ $\qquad x = \frac{\pi}{2} \Rightarrow u = 2$

$$\therefore\ I = \int_0^2 \sqrt{u}\,du$$

$$= \frac{2}{3}\left[u^{3/2}\right]_0^2 = \boxed{\frac{2}{3}\,2^{\frac{3}{2}}}$$

Question 2 [10 points]: Evaluate

$$I = \int t^{-1/2}(\ln t)^2\,dt$$

$$u = (\ln t)^2 \qquad dv = t^{-\frac{1}{2}}dt$$

$$du = 2\ln t\left(\tfrac{1}{t}\right)dt \qquad v = 2t^{\frac{1}{2}}$$

$$\therefore\ I = \int u\,dv = uv - \int v\,du$$

$$= (\ln t)^2 \cdot 2t^{1/2} - \int 2t^{1/2} \cdot 2\ln t \cdot \frac{1}{t}\,dt$$

$$= 2t^{1/2}(\ln t)^2 - 4\underbrace{\int t^{-1/2}\ln t\,dt}_{\substack{u=\ln t \quad dv = t^{-\frac{1}{2}}dt \\ du = \frac{1}{t}dt \quad v = 2t^{1/2}}}$$

$$= 2t^{1/2}(\ln t)^2 - 4\left[(\ln t)\cdot 2t^{1/2} - \int 2t^{1/2}\frac{1}{t}\,dt\right]$$

$$= 2t^{1/2}(\ln t)^2 - 8t^{1/2}\ln t + 8\int t^{-1/2}\,dt$$

$$= 2t^{1/2}(\ln t)^2 - 8t^{1/2}\ln t + 8\cdot 2t^{1/2} + C$$

$$= \boxed{2t^{1/2}(\ln t)^2 - 8t^{1/2}(\ln t) + 16t^{1/2} + C}$$

Question 3 [10 points]: Evaluate

$$I = \int_0^{\pi/4} \sec^4\theta \tan^4\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \sec^2\theta \tan^4\theta \sec^2\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} (1+\tan^2\theta) \tan^4\theta \sec^2\theta \, d\theta$$

let $u = \tan\theta$ $\quad\quad$ $\theta = 0 \Rightarrow u = 0$

$du = \sec^2\theta \, d\theta$ $\quad\quad$ $\theta = \frac{\pi}{4} \Rightarrow u = 1$

$$\therefore I = \int_0^1 (1+u^2)u^4 \, du$$

$$= \int_0^1 u^4 + u^6 \, du$$

$$= \left[\frac{u^5}{5} + \frac{u^7}{7}\right]_0^1$$

$$= \left(\frac{1}{5} + \frac{1}{7}\right) - (0 + 0)$$

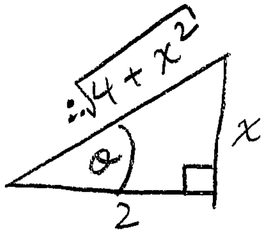
$$= \boxed{\frac{12}{35}}$$

Question 4 [10 points]: Evaluate

$$I = \int \frac{x^3}{\sqrt{4+x^2}}\,dx$$

let $x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$



$$\therefore I = \int \frac{2^3\tan^3\theta \cdot 2\sec^2\theta}{\sqrt{4+4\tan^2\theta}}\,d\theta$$

$$= \frac{2^4}{2}\int \frac{\tan^3\theta\sec^2\theta}{\cancel{\sec\theta}}\,d\theta$$

$$= 8\int \tan^2\theta \quad \sec\theta\tan\theta\, d\theta$$

$$= 8\int (\sec^2\theta - 1)\sec\theta\tan\theta\, d\theta$$

let $u = \sec\theta$

$du = \sec\theta\tan\theta\, d\theta$

$$\therefore I = 8\int (u^2-1)\,du$$

$$= 8\left[\frac{u^3}{3} - u\right] + C$$

$$= 8\left[\frac{\sec^3\theta}{3} - \sec\theta\right] + C$$

$$= \frac{8}{3}\left(\frac{\sqrt{4+x^2}}{2}\right)^3 - 8\frac{\sqrt{4+x^2}}{2} + C = \boxed{\frac{1}{3}\left(\sqrt{4+x^2}\right)^3 - 4\sqrt{4+x^2} + C}$$

Question 5 [10 points]: Evaluate

$$I = \int \frac{x^2 - 8x + 18}{x(x-3)^2}\,dx$$

$$\frac{x^2 - 8x + 18}{x(x-3)^2} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

$$= \frac{A(x-3)^2 + Bx(x-3) + Cx}{x(x-3)^2}$$

$$= \frac{Ax^2 - 6Ax + 9A + Bx^2 - 3Bx + Cx}{x(x-3)^2}$$

$$= \frac{(A+B)x^2 + (-6A-3B+C)x + 9A}{x(x-3)^2}$$

$\therefore$ $A + B = 1$ ①

$-6A - 3B + C = -8$ ②

$9A = 18$ ③

$\therefore$ ③ $\Rightarrow$ $A = 2$

① $\Rightarrow$ $B = 1 - A = 1 - 2 = -1$

② $\Rightarrow$ $C = -8 + 6A + 3B$

$= -8 + 12 - 3$

$= 1$

$$\therefore I = \int \frac{2}{x} - \frac{1}{x-3} + \frac{1}{(x-3)^2}\,dx$$

$$= \boxed{2\ln|x| - \ln|x-3| - \frac{1}{x-3} + C}$$