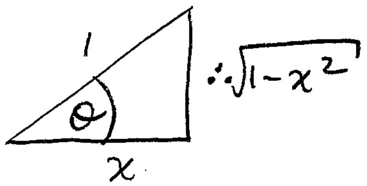
**Question 1:**

**(a)[3 points]** Simplify $\tan(\cos^{-1} x)$. Your answer should not contain any trigonometric functions.

Let $\theta = \cos^{-1}\left(\frac{x}{1}\right)$

$\therefore \cos\theta = \frac{x}{1}$




$$\therefore \tan(\cos^{-1} x) = \tan\theta = \boxed{\frac{\sqrt{1-x^2}}{x}}$$

**(b)[4 points]** Find and simplify the derivative of

$$\arctan(t) + \arctan(1/t)$$

$$\frac{d}{dt}\left[\arctan(t) + \arctan\left(\frac{1}{t}\right)\right]$$

$$= \frac{1}{1+t^2} + \frac{1}{1+\left(\frac{1}{t}\right)^2} \cdot \left(\frac{-1}{t^2}\right)$$

$$= \frac{1}{1+t^2} - \frac{1}{t^2+1}$$

$$= \boxed{0}$$

**(b)[3 points]** Find $\lim_{x\to\infty} \sin^{-1}\left(1 - \frac{1}{\sqrt{x}}\right)$

As $x \to \infty$, $1 - \frac{1}{\sqrt{x}} \to 1^-$,

$$\therefore \lim_{x\to\infty} \sin^{-1}\left(1 - \frac{1}{\sqrt{x}}\right) = \boxed{\frac{\pi}{2}}$$

**Question 2:**

**(a)[5 points]** Solve for $x$:

$$\cosh(x) = 1 + \sinh(x)$$

$$\frac{e^x + e^{-x}}{2} = 1 + \frac{e^x - e^{-x}}{2}$$

$$\cancel{e^x} + e^{-x} = 2 + \cancel{e^x} - e^{-x}$$

$$2e^{-x} = 2$$

$$e^{-x} = 1$$

$$\boxed{x = 0}$$

**(b)[5 points]** Let $f(x) = x\cosh(x^2)$. Compute $f'(0)$.

$$f'(x) = 1 \cdot \cosh(x^2) + x\sinh(x^2) \cdot 2x$$

$$f'(0) = 1 \cdot 1 + 0$$

$$= \boxed{1}$$

**Question 3:**

**(a)[5 points]** Evaluate the limit if it exists:

$$\lim_{x\to 1} \frac{\ln x}{\sin(\pi x)}. \quad \sim \text{"}\frac{0}{0}\text{"}$$

$$\overset{H}{=} \lim_{x\to 1} \frac{\frac{1}{x}}{\pi\cos(\pi x)}$$

$$= \frac{\frac{1}{1}}{\pi(-1)}$$

$$= \boxed{\frac{-1}{\pi}}$$

**(b)[5 points]** Evaluate the limit if it exists:

$$\lim_{x\to 0^+} x^{(x^5)}. \quad \sim \text{"}0^0\text{"}$$

$$x^{(x^5)} = e^{x^5\ln x}$$

$$\lim_{x\to 0^+} x^5 \ln x$$

$$= \lim_{x\to 0^+} \frac{\ln x}{x^{-5}} \sim \text{"}\frac{-\infty}{\infty}\text{"}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-5x^{-6}}$$

$$= \lim_{x\to 0^+} \frac{x^5}{-5}$$

$$= 0$$

$$\therefore \lim_{x\to 0^+} x^{(x^5)} = e^0 = \boxed{1}$$

**Question 4:**

**(a)**[**5 points**] Suppose $f''(t) = e^t + 2t$ and that $f(0) = 1$, $f'(0) = 2$. Find $f(t)$.

$$f'(t) = e^t + \frac{2t^2}{2} + C_1 = e^t + t^2 + C_1$$

$$\therefore f(t) = e^t + \frac{t^3}{3} + C_1 t + C_2$$

$$f(0) = 1 \Rightarrow 1 + 0 + 0 + C_2 = 1 \Rightarrow C_2 = 0$$

$$f'(0) = 2 \Rightarrow 1 + 0 + C_1 = 2 \Rightarrow C_1 = 1$$

$$\therefore \boxed{f(t) = e^t + \frac{t^3}{3} + t}$$

**(b)**[**5 points**] Let $s(t)$ be the displacement in metres of a particle at time $t$ seconds, $v(t)$ be its velocity, and $a(t)$ its acceleration. If

$$a(t) = \cos t + \sin t, \quad s(0) = 0, \quad v(0) = 5\ ,$$

find the displacement of the particle at time $t = \pi$ seconds.

$$v(t) = \sin t - \cos t + C_1$$

$$s(t) = -\cos t - \sin t + C_1 t + C_2$$

$$s(0) = 0 \Rightarrow -1 - 0 + 0 + C_2 = 0 \Rightarrow C_2 = 1$$

$$v(0) = 5 \Rightarrow 0 - 1 + C_1 = 5 \Rightarrow C_1 = 6$$

$$\therefore s(t) = -\cos t - \sin t + 6t + 1$$

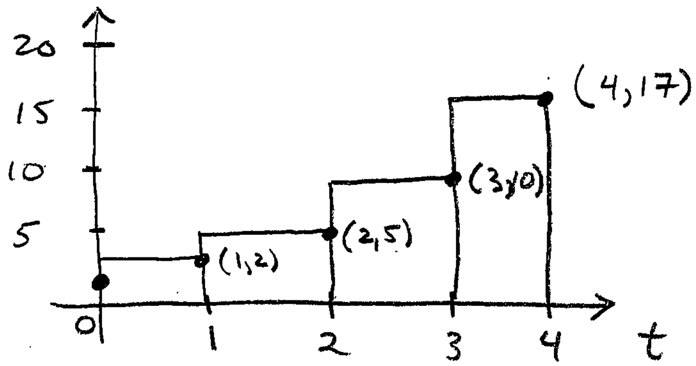
$$s(\pi) = -\cos(\pi) - \cancel{\sin(\pi)}^{0} + 6\pi + 1$$

$$= 1 + 6\pi + 1$$

$$= \boxed{6\pi + 2 \text{ m}}$$

**Question 5:**

**(a)**[**5 points**] The rabbit population on campus is increasing at a rate of $r(t) = 1 + t^2$ rabbits per week, where $t = 0$ corresponds to the present. Use $R_4$ to estimate the total increase in the rabbit population over the next four weeks. A sketch of the graph of $y = r(t)$ would be helpful.




Increase in pop. $\approx R_4 = (2)(1) + (5)(1) + (10)(1) + (17)(1)$

$= \boxed{34 \text{ rabbits.}}$

**(b)**[**5 points**] The limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(i\frac{\pi}{n}\right)\frac{\pi}{n}$$

represents the area between the graph of a certain function and the $x$-axis. Draw the graph and shade the area in question. To get full marks you must correctly identify the function and the interval over which the area is measured.

Comparing $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \sin\left(i\frac{\pi}{n}\right)\frac{\pi}{n} = \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} f(x_i)\,\Delta x$,

$\Delta x = \frac{\pi}{n} = \frac{b-a}{n}$ ; $x_i = i\Delta x = i\frac{\pi}{n}$, $f(x) = \sin(x)$,

$\therefore$ area is