Question 1:

(a)[3 points] Simplify $\tan(\cos^{-1} x)$. Your answer should not contain any trigonometric functions.

(b)[4 points] Find and simplify the derivative of

$$\arctan(t) + \arctan(1/t)$$

(b)[3 points] Find $\lim_{x\to\infty} \sin^{-1}\left(1 - \frac{1}{\sqrt{x}}\right)$

Question 2:

(a)[5 points] Solve for x:

$$\cosh(x) = 1 + \sinh(x)$$

(b)[5 points] Let $f(x) = x\cosh(x^2)$. Compute $f'(0)$.

Question 3:

(a)[5 points] Evaluate the limit if it exists:

$$\lim_{x \to 1} \frac{\ln x}{\sin(\pi x)} \ .$$

(b)[5 points] Evaluate the limit if it exists:

$$\lim_{x \to 0^+} x^{(x^5)} \ .$$

Question 4:

(a)[5 points] Suppose $f''(t) = e^t + 2t$ and that $f(0) = 1$, $f'(0) = 2$. Find $f(t)$.

(b)[5 points] Let $s(t)$ be the displacement in metres of a particle at time t seconds, $v(t)$ be its velocity, and $a(t)$ its acceleration. If

$$a(t) = \cos t + \sin t, \qquad s(0) = 0, \qquad v(0) = 5 \ ,$$

find the displacement of the particle at time $t = \pi$ seconds.

Question 5:

(a)[5 points] The rabbit population on campus is increasing at a rate of $r(t) = 1 + t^2$ rabbits per week, where $t = 0$ corresponds to the present. Use R_4 to estimate the total increase in the rabbit population over the next four weeks. A sketch of the graph of $y = r(t)$ would be helpful.

(b)[5 points] The limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(i\frac{\pi}{n}\right)\frac{\pi}{n}$$

represents the area between the graph of a certain function and the x-axis. Draw the graph and shade the area in question. To get full marks you must correctly identify the function and the interval over which the area is measured.