

Question 1:

(a) [7 points] Find the linear approximation to $f(x) = \sqrt{25 - x^2}$ at $a = 3$.

$$f(x) = \sqrt{25 - x^2} \quad ; \quad f(3) = \sqrt{25 - 3^2} = 4$$

$$f'(x) = \frac{1}{2} (25 - x^2)^{-\frac{1}{2}} (-2x) \quad ; \quad f'(3) = \frac{-3}{\sqrt{25 - 3^2}} = \frac{-3}{4}$$

$$\begin{aligned} \therefore L(x) &= f(a) + f'(a)(x-a) \\ &= 4 - \frac{3}{4}(x-3) \end{aligned}$$

(b) [3 points] Use your result from (a) to approximate $\sqrt{21}$.

$$\begin{aligned} \sqrt{21} &= f(2) \approx L(2) = 4 - \frac{3}{4}(2-3) \\ &= 4 + \frac{3}{4} \\ &= \frac{19}{4} \quad \text{or} \quad 4.75 \end{aligned}$$

Question 2:

(a)[5 points] Solve for x :

$$\ln(1 + e^{-x}) = 3$$

$$1 + e^{-x} = e^3$$

$$e^{-x} = e^3 - 1$$

$$-x = \ln(e^3 - 1)$$

$$x = -\ln(e^3 - 1)$$

(b)[5 points] Evaluate the limit

$$\lim_{x \rightarrow \infty} \frac{3e^{2x} - 2e^{-3x}}{2e^{2x} + 3e^{-3x} - 1} \quad \begin{array}{l} \div e^{2x} \\ \div e^{2x} \end{array}$$

$$= \lim_{x \rightarrow \infty} \frac{3 - 2e^{-5x}}{2 + 3e^{-5x} - e^{-2x}}$$

$$= \frac{3}{2}$$

Question 3:

(a)[3 points] Differentiate

$$y = e^{-5x} \cos(7x)$$

$$y' = e^{-5x} (-5) \cos(7x) - e^{-5x} \sin(7x) \cdot 7$$

(b)[3 points] Differentiate

$$y = \log_5(xe^x)$$

$$y' = \frac{1}{xe^x \cdot \ln 5} \cdot [e^x + xe^x]$$

(c)[4 points] Use logarithmic differentiation to find $\frac{dy}{dx}$:

$$y = \frac{x^{\cos x}}{\tan^7 x}$$

$$\ln y = \cos x \ln x - 7 \ln(\tan x)$$

$$\therefore \frac{1}{y} y' = -\sin x \cdot \ln x + \cos x \cdot \frac{1}{x} - 7 \cdot \frac{1}{\tan x} \cdot \sec^2 x$$

$$\therefore y' = \left[-\sin x \cdot \ln x + \cos x \cdot \frac{1}{x} - 7 \cdot \frac{1}{\tan x} \cdot \sec^2 x \right] \left(\frac{x^{\cos x}}{\tan^7 x} \right)$$

Question 4 [10 points]: Find the absolute maximum and absolute minimum values of $f(x) = \frac{4x}{x^2+4}$ on the interval $[0, 5]$.

$\underbrace{\hspace{2cm}}$
closed

$\underbrace{\hspace{2cm}}$
continuous

$$f(x) = \frac{4x}{x^2+4}$$

$$f'(x) = \frac{(x^2+4)(4) - (4x)(2x)}{(x^2+4)^2} = \frac{16 - 4x^2}{(x^2+4)^2}$$
$$= \frac{4(2-x)(2+x)}{(x^2+4)^2}$$

$f'(x) = 0$ at $x = 2, -2$, but only $x = 2$ is in $[0, 5]$.

x	$f(x) = \frac{4x}{x^2+4}$
0	0
2	1
5	$\frac{20}{29}$

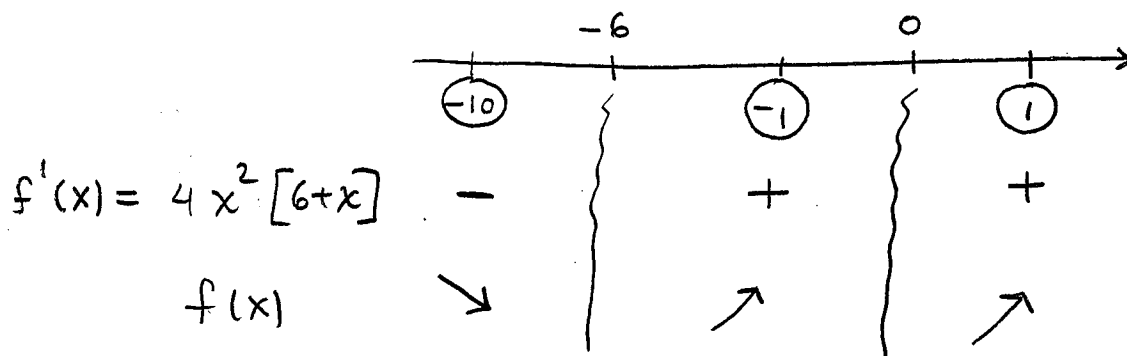
\therefore the absolute max. of f is 1,
the absolute min. of f is 0.
(on $[0, 5]$),

Question 5: For this question use the function $f(x) = 200 + 8x^3 + x^4$.

(a)[5 points] Find the intervals of increase and decrease of f .

$$f'(x) = 24x^2 + 4x^3 = 4x^2 [6+x]$$

$f'(x) = 0$ at $x=0, -6$ (there are no other critical numbers.)

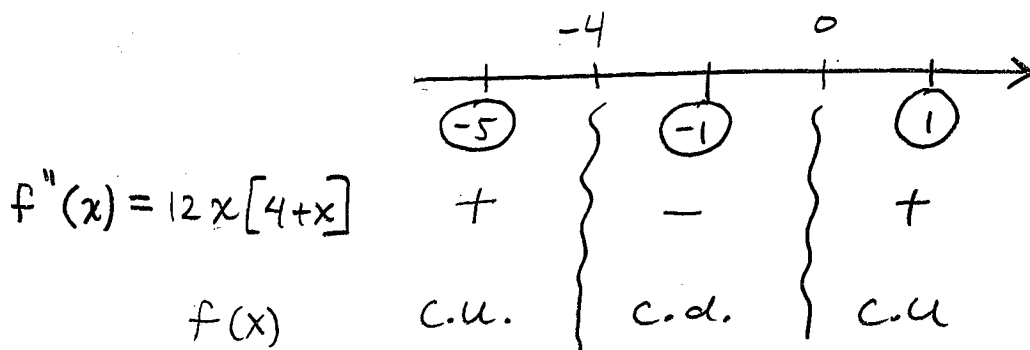


$\therefore f$ is decreasing on $(-\infty, -6)$, increasing on $(-6, \infty)$.

(b)[5 points] Find the intervals of concavity of the graph of f .

$$f''(x) = 48x + 12x^2 = 12x [4+x]$$

$f''(x) = 0$ at $x=0, x=-4$.



\therefore Graph of f is concave up on $(-\infty, -4) \cup (0, \infty)$, concave down on $(-4, 0)$.