Question 1:

(a)[7 points] Find the linear approximation to $f(x) = \sqrt{25 - x^2}$ at $a = 3$.

$f(x) = \sqrt{25-x^2}$; $f(3) = \sqrt{25-3^2} = 4$

$f'(x) = \frac{1}{\cancel{2}}(25-x^2)^{-\frac{1}{2}}(-\cancel{2}x)$; $f'(3) = \frac{-3}{\sqrt{25-3^2}} = \frac{-3}{4}$

$\therefore \; L(x) = f(a) + f'(a)(x-a)$

$= 4 - \frac{3}{4}(x-3)$

(b)[3 points] Use your result from (a) to approximate $\sqrt{21}$.

$\sqrt{21} = f(2) \approx L(2) = 4 - \frac{3}{4}(2-3)$

$= 4 + \frac{3}{4}$

$= \frac{19}{4}$ or 4.75

Question 2:

(a)[5 points] Solve for x:

$$\ln\left(1+e^{-x}\right) = 3$$

$$1+e^{-x} = e^{3}$$

$$e^{-x} = e^{3}-1$$

$$-x = \ln(e^{3}-1)$$

$$x = -\ln(e^{3}-1)$$

(b)[5 points] Evaluate the limit

$$\lim_{x\to\infty} \frac{3e^{2x}-2e^{-3x}}{2e^{2x}+3e^{-3x}-1} \quad \begin{matrix} \div e^{2x} \\ \div e^{2x} \end{matrix}$$

$$= \lim_{x\to\infty} \frac{3-2e^{-5x}}{2+3e^{-5x}-e^{-2x}}$$

$$= \frac{3}{2}$$

Question 3:

(a)[**3 points**] Differentiate

$$y = e^{-5x}\cos(7x)$$

$$y' = e^{-5x}(-5)\cos(7x) - e^{-5x}\sin(7x)\cdot 7$$

(b)[**3 points**] Differentiate

$$y = \log_5(xe^x)$$

$$y' = \frac{1}{xe^x \cdot \ln 5} \cdot \left[e^x + xe^x\right]$$

(c)[**4 points**] Use logarithmic differentiation to find $\frac{dy}{dx}$:

$$y = \frac{x^{\cos x}}{\tan^7 x}$$

$$\ln y = \cos x \ln x - 7\ln(\tan x)$$

$$\therefore \frac{1}{y}y' = -\sin x \cdot \ln x + \cos x \cdot \frac{1}{x} - 7 \cdot \frac{1}{\tan x} \cdot \sec^2 x$$

$$\therefore y' = \left[-\sin x \cdot \ln x + \cos x \cdot \frac{1}{x} - 7 \cdot \frac{1}{\tan x} \cdot \sec^2 x\right]\left(\frac{x^{\cos x}}{\tan^7 x}\right)$$

Question 4 [10 points]: Find the absolute maximum and absolute minimum values of $f(x) = \underbrace{\dfrac{4x}{x^2+4}}_{\text{continuous}}$ on the interval $\underbrace{[0,5]}_{\text{closed}}$.

$$f(x) = \frac{4x}{x^2+4}$$

$$f'(x) = \frac{(x^2+4)(4) - (4x)(2x)}{(x^2+4)^2} = \frac{16 - 4x^2}{(x^2+4)^2}$$

$$= \frac{4(2-x)(2+x)}{(x^2+4)^2}$$

$f'(x) = 0$ at $x = 2, -2$, but only $x = 2$ is in $[0,5]$.

x	$f(x) = \frac{4x}{x^2+4}$
0	0
2	1
5	$\frac{20}{29}$

$\therefore$ the absolute max. of f is 1,
the absolute min. of f is 0.
(on $[0,5]$).

Question 5: For this question use the function $f(x) = 200 + 8x^3 + x^4$.

(a)[5 points] Find the intervals of increase and decrease of f.

$$f'(x) = 24x^2 + 4x^3 = 4x^2[6+x]$$

$f'(x) = 0$ at $x = 0, -6$ (there are no other critical numbers.).

	-10	-6	-1	0	1
$f'(x) = 4x^2[6+x]$	$-$		$+$		$+$
$f(x)$	$\searrow$		$\nearrow$		$\nearrow$

$\therefore$ f is decreasing on $(-\infty, -6)$, increasing on $(-6, \infty)$.

(b)[5 points] Find the intervals of concavity of the graph of f.

$$f''(x) = 48x + 12x^2 = 12x[4+x]$$

$f''(x) = 0$ at $x = 0$, $x = -4$.

	-5	-4	-1	0	1
$f''(x) = 12x[4+x]$	$+$		$-$		$+$
$f(x)$	c.u.		c.d.		c.u

$\therefore$ Graph of f is concave up on $(-\infty, -4)$ & $(0, \infty)$, concave down on $(-4, 0)$.