Question 1:

(a)[7 points] Use the limit definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{2x}{x+1}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{2(x+h)}{x+h+1} - \frac{2x}{x+1}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{2(x+h)(x+1) - 2x(x+h+1)}{(x+h+1)(x+1)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\cancel{2x^2} + \cancel{2hx} + \cancel{2x} + 2h - \cancel{2x^2} - \cancel{2xh} - \cancel{2x}}{(x+h+1)(x+1)}\right]$$

$$= \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}(x+h+1)(x+1)}$$

$$= \boxed{\frac{2}{(x+1)^2}}$$

(b)[3 points] Now use your derivative rules to check your answer in part (a).

$$f'(x) = \frac{(x+1)(2) - (2x)(1)}{(x+1)^2}$$

$$= \frac{2x + 2 - 2x}{(x+1)^2}$$

$$= \boxed{\frac{2}{(x+1)^2}}$$

Question 2:

(a)[3 points] Let $f(x) = 5x^3 - 7x + 11$. Find $f'(x)$.

$$f'(x) = 15x^2 - 7$$

(b)[3 points] Find $\frac{dy}{dx}$ if $y = 2\sqrt{x}\cos x$. $= 2\ x^{1/2} \cos x$

$$\frac{dy}{dx} = 2\left[\frac{1}{2}x^{-\frac{1}{2}}\cos x - x^{\frac{1}{2}}\sin x\right]$$

$$= \frac{\cos x}{\sqrt{x}} - 2\sqrt{x}\sin x$$

(c)[4 points] Let $g(t) = \frac{1 + \sin t}{t + \cos t}$. Find $g'(0)$.

$$g'(t) = \frac{(t + \cos t)(\cos t) - (1 + \sin t)(1 - \sin t)}{(t + \cos t)^2}$$

$$g'(0) = \frac{(0 + 1)(1) - (1 + 0)(1 - 0)}{(0 + 1)^2}$$

$$= 0$$

Question 3:

(a)[3 points] Find $f'(x)$ if $f(x) = \sqrt[3]{1 + \tan x}$. $= (1+\tan x)^{\frac{1}{3}}$

$$f'(x) = \frac{1}{3}(1+\tan x)^{-\frac{2}{3}} \sec^2 x$$

(b)[3 points] Find $\frac{dy}{dx}$ if $y = \sin\sqrt{2 - x^2}$. $= \sin\left((2-x^2)^{1/2}\right)$

$$\frac{dy}{dx} = \cos\left((2-x^2)^{1/2}\right) \cdot \frac{1}{2}(2-x^2)^{-\frac{1}{2}}(-2x)$$

$$= \frac{-x\cos\left(\sqrt{2-x^2}\right)}{\sqrt{2-x^2}}$$

(c)[4 points] Find $f''(1)$ if $f(t) = \sqrt{8t^2 + 1}$. $= (8t^2+1)^{1/2}$

$$f'(t) = \frac{1}{2}(8t^2+1)^{-\frac{1}{2}}(16t) = 8t(8t^2+1)^{-\frac{1}{2}}$$

$$f''(t) = 8(8t^2+1)^{-\frac{1}{2}} + 8t\left(\frac{-1}{2}\right)(8t^2+1)^{-3/2}(16t)$$

$$= 8(8t^2+1)^{-\frac{1}{2}} - (8t)^2(8t^2+1)^{-3/2}$$

$$\therefore f''(1) = 8(8+1)^{-\frac{1}{2}} - 8^2(8+1)^{-\frac{3}{2}}$$

$$= \frac{8}{3} - \frac{64}{27} = \frac{72-64}{27} = \boxed{\frac{8}{27}}$$

Question 4:

(a)[**5 points**] Find the equation of the tangent line to the curve

$$y^2(y^2-4) = x^2(x^2-5)$$

at the point $(0,-2)$.

$$\frac{d}{dx}\left[y^2(y^2-4)\right] = \frac{d}{dx}\left[x^2(x^2-5)\right]$$

$$2yy'(y^2-4) + y^2 2yy' = 2x(x^2-5) + x^2 \cdot 2x$$

at $x=0,\ y=-2$:

$$2(-2)y'\cancel{((-2)^2-4)} + (-2)^2 \cdot 2 \cdot (-2)y' = 2 \cdot 0 \cdot \cancel{(0^2-5)} + 0^2 \cdot \cancel{2 \cdot 0}$$

$0 \qquad 0 \qquad 0$

$$\therefore -16y' = 0$$

$$\therefore y' = 0$$

$\therefore$ Equation of tangent line is $\boxed{y = -2}$

(b)[**5 points**] A particle moving in a straight line has position $s(t)$ metres after t seconds, where $s(t) = 6t^3 - 3t^2 + k$ where k is some positive constant. At what time(s) is the particle's acceleration zero?

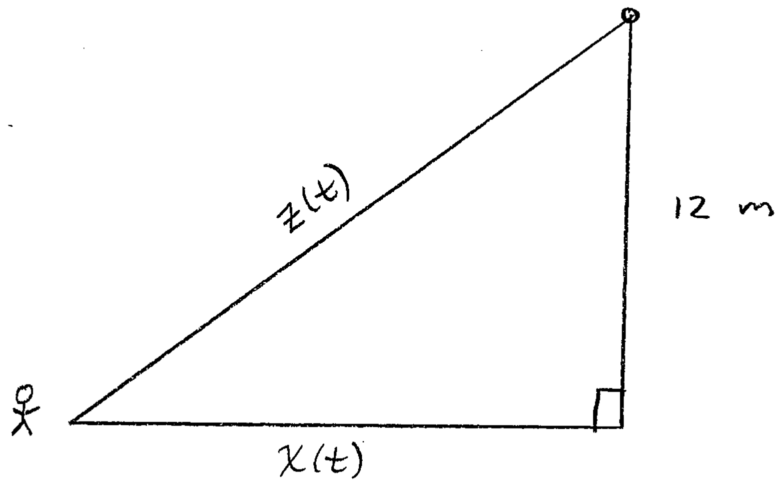
$$s'(t) = 18t^2 - 6t$$

$$s''(t) = 36t - 6$$

$$s''(t) = 0 \Rightarrow 36t - 6 = 0$$

$$t = \frac{6}{36} = \boxed{\frac{1}{6}\ s}$$

Question 5 [10 points] A person is walking at 2 metres per second toward the bottom of a pole 12 metres in height. How fast is the distance between the person and the top of the pole changing when the person is 5 metres from the bottom of the pole? State units with your answer.



$$\frac{dx}{dt} = -2\ \frac{m}{s}.$$

Find $\frac{dz}{dt}$ when $x = 5$ m.

$$z = \sqrt{x^2+12^2} = (x^2+12^2)^{\frac{1}{2}}$$

$$\frac{dz}{dt} = \frac{1}{2}(x^2+12^2)^{-\frac{1}{2}}\left(2x\frac{dx}{dt}\right)$$

when $x=5$:

$$\frac{dz}{dt} = \frac{1}{\cancel{2}}(5^2+12^2)^{-\frac{1}{2}}\left(\cancel{2}\cdot 5\cdot(-2)\right)$$

$$= \frac{-10}{\sqrt{169}} = \boxed{\frac{-10}{13}\ \frac{m}{s}}$$