

Question 1:

(a)[7 points] Let $f(x) = \frac{1}{x+1}$. Evaluate and simplify the difference quotient

$$\frac{f(x+h) - f(x)}{h}.$$

(b)[3 points] Find the domain of the function $f(x) = \frac{1}{\sqrt{x^2 - 4}}$.

Question 2:

(a)[3 points] Let $f(x) = \sqrt{x+3}$ and $g(x) = x^2 - 3$. Find $g \circ f$ and state the domain.

(b)[4 points] Again let $f(x) = \sqrt{x+3}$ and $g(x) = x^2 - 3$. Find $\frac{f}{g}$ and state the domain.

(c)[3 points] Let $F(x) = 1 + \sqrt{\sin x}$. Find functions f and g so that $F = f \circ g$.

Question 3:

(a)[5 points] Evaluate the limit if it exists:

$$\lim_{x \rightarrow 4} \frac{x^2 - 4x}{x^2 - 3x - 4}.$$

(b)[5 points] Evaluate the limit if it exists:

$$\lim_{h \rightarrow 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}.$$

Question 4:

(a)[5 points] Evaluate the limit if it exists:

$$\lim_{x \rightarrow 0} x^2 \cos \left(\frac{1 + x^2}{x^2 - x} \right).$$

(b)[5 points] Evaluate the limit if it exists:

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta + \tan \theta}$$

Question 5:

(a)[5 points] Let

$$g(x) = \begin{cases} x^4 - cx^2 & \text{if } x < 2 \\ c^2x + 18 & \text{if } x \geq 2 \end{cases}$$

Find the constant c that makes g continuous at all real numbers.

(b)[5 points] Evaluate the limit if it exists:

$$\lim_{x \rightarrow \infty} \sqrt{x^2 - 3x} - \sqrt{x^2 - 5x} .$$