

(1)[5 points] Find a formula for the inverse of $y = \ln(x+3)$.

$$y = \ln(x+3)$$

$$e^y = x+3$$

$$x = e^y - 3$$

$$x \leftrightarrow y : \boxed{y = e^x - 3}$$

$$\text{or } f^{-1}(x) = e^x - 3$$

(2)[5 points] Express as a single logarithm:

$$\ln(1+x^2) + \frac{1}{2} \ln x - \ln(\sin x)$$

$$= \ln(1+x^2) + \ln x^{1/2} - \ln(\sin x)$$

$$= \ln \left[\frac{(1+x^2) x^{1/2}}{\sin x} \right]$$

(3)[5 points] Find the limit

$$\lim_{x \rightarrow \infty} [\ln(1+x^2) - \ln(1+x)]$$

$$= \lim_{x \rightarrow \infty} \ln \left(\frac{1+x^2}{1+x} \right)$$

$$= \lim_{x \rightarrow \infty} \ln \left(\frac{\frac{1}{x} + x}{\frac{1}{x} + 1} \right)$$

$\underbrace{\hspace{10em}}_{\rightarrow \infty}$

$$= \infty$$