Math 121 Sec F08N05 - Quiz 5 Name: SOLUTIONS.
Oct 31 2008 Stu#:

(1)[**5 points**] Use a linear approximation to estimate $(8.06)^{2/3}$.

Here $f(x) = x^{2/3}$, $a = 8$ and we wish to estimate $f(8.06)$.

$$f(a) = 8^{2/3} = (8^{1/3})^2 = 2^2 = 4$$

$$f'(a) = \frac{2}{3}x^{-1/3}\Big|_{x=8} = \frac{2}{3}\frac{1}{8^{1/3}} = \frac{1}{3}$$

$$\therefore L(x) = f(a) + f'(a)(x-a)$$

$$= 4 + \frac{1}{3}(x-8)$$

$$\therefore f(8.06) \approx L(8.06) = 4 + \frac{1}{3}(8.06-8)$$

$$= 4 + \frac{0.06}{3}$$

$$= \boxed{4.02}$$

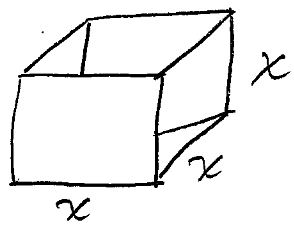
(2)[**5 points**] Compute the differential dy of $y = x^2 \sin(2x)$.

$$dy = \frac{d}{dx}\left[x^2 \sin(2x)\right] dx$$

$$= \left[(2x)\sin(2x) + x^2\cos(2x)\cdot 2\right] dx$$

$$= (2x)\left(\sin(2x) + x\cos(2x)\right) dx$$

(3)[5 points] The edge of a cube was found to be 30 cm with a possible error in measurement of 0.1 cm. Use differentials to estimate the maximum possible error and relative error (as a percentage) in computing the volume of the cube.



$V = x^3$

$dx = \Delta x = 0.1$ cm

$$\Delta V \approx dV = \frac{d}{dx}[x^3]\,dx$$

$$= 3x^2 dx$$

$\therefore$ with $x = 30$ cm, $dx = 0.1$ cm:

$\therefore$ max. error $\Delta V \approx dV = 3(30)^2(0.1) = \boxed{270 \text{ cm}^3}$

relative error $\dfrac{\Delta V}{V} \approx \dfrac{dV}{V} = \dfrac{3x^2 dx}{x^3} = \dfrac{3}{x} dx.$

At $x = 30$ cm, $dx = 0.1$ cm;

$$\frac{\Delta V}{V} \approx \frac{3}{30} \cdot (0.1) = 0.01 = \boxed{1\%}$$