

(1)[5 points] Find the derivative of $G(t) = \frac{4t}{t+1}$ using the definition of the derivative. State the domains of both $G(t)$ and $G'(t)$.

$$G'(t) = \lim_{h \rightarrow 0} \frac{G(t+h) - G(t)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{4(t+h)}{t+h+1} - \frac{4t}{t+1} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{4(t+h)(t+1) - 4t(t+h+1)}{(t+1)(t+h+1)} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{\cancel{4t^2} + 4ht + \cancel{4t} + 4h - \cancel{4t^2} - \cancel{4ht} - 4t}{(t+1)(t+h+1)} \right]$$

$$= \frac{4}{(t+1)^2}$$

domain of $G(t) = \frac{4t}{t+1}$ is $(-\infty, -1) \cup (-1, \infty)$;

domain of $G'(t) = \frac{4}{(t+1)^2}$ is also $(-\infty, -1) \cup (-1, \infty)$.

(2)[5 points] Differentiate

$$y = \frac{x^2 + 4x + 3}{\sqrt{x}}$$

$$y = \frac{x^2}{x^{1/2}} + \frac{4x}{x^{1/2}} + \frac{3}{x^{1/2}}$$

$$y = x^{3/2} + 4x^{1/2} + 3x^{-1/2}$$

$$y' = \frac{3}{2}x^{1/2} + 4 \cdot \frac{1}{2}x^{-1/2} + 3\left(-\frac{1}{2}\right)x^{-3/2}$$

$$= \frac{3}{2}x^{1/2} + 2x^{-1/2} - \frac{3}{2}x^{-3/2}$$

(3)[5 points] The equation of motion of a particle is $s = t^3 - 3t$, where s is in metres and t is in seconds. Find the acceleration after 2 seconds. Include units in your answer.

$$s' = 3t^2 - 3 \quad \frac{m}{s}$$

$$s'' = 6t \quad \frac{m}{s^2}$$

$$\therefore s'' \Big|_{t=2} = 6(2) = 12 \frac{m}{s^2}$$