Math 121 Sec F08N05 - Quiz 3 Name: SOLUTIONS.
Oct 3 2008 Stu#:

(1)[8 points] Let $f(x) = 3x^2 - 5x$. Find the equation of the tangent line to the graph of $y = f(x)$ at the point $(2, 2)$.

$$m = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{[3(2+h)^2 - 5(2+h)] - [3(2)^2 - 5(2)]}{h}$$

$$= \lim_{h \to 0} \frac{3(4 + 4h + h^2) - 10 - 5h - 12 + 10}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{12} + 12h + 3h^2 \cancel{- 10} - 5h \cancel{- 12} \cancel{+ 10}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(7 + 3h)}{\cancel{h}}$$

$$= 7$$

$\therefore$ Equation of line is

$$y - y_0 = m(x - x_0)$$

$$\boxed{y - 2 = 7(x - 2)}$$

or $\boxed{y = 7x - 12}$

(2)[**7 points**] Let $f(x) = \dfrac{1}{\sqrt{x+2}}$. Find $f'(a)$.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{1}{\sqrt{a+h+2}} - \dfrac{1}{\sqrt{a+2}}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sqrt{a+2}-\sqrt{a+h+2}}{\sqrt{a+h+2}\sqrt{a+2}} \cdot \frac{\sqrt{a+2}+\sqrt{a+h+2}}{\sqrt{a+2}+\sqrt{a+h+2}}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\cancel{a}+\cancel{2}-\cancel{a}-h-\cancel{2}}{\sqrt{a+h+2}\sqrt{a+2}\left(\sqrt{a+2}+\sqrt{a+h+2}\right)}\right]$$

$$= \lim_{h\to 0} \frac{1}{\cancel{h}}\left[\frac{-\cancel{h}}{\sqrt{a+h+2}\sqrt{a+2}\left(\sqrt{a+2}+\sqrt{a+h+2}\right)}\right]$$

$$= \frac{-1}{\sqrt{a+2}\sqrt{a+2}\left(\sqrt{a+2}+\sqrt{a+2}\right)}$$

$$= \frac{-1}{2\sqrt{a+2}\,(a+2)}$$

$$= \boxed{\frac{-1}{2(a+2)^{3/2}}}$$