

(1)[5 points] Let $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$. Find and simplify $f \circ g$ and state the domain.

$$(f \circ g)(x) = f(g(x))$$

$$\begin{aligned}
 &= \left(\frac{x+1}{x+2} \right) + \frac{1}{\left(\frac{x+1}{x+2} \right)} && \left. \begin{array}{l} \text{domain:} \\ \text{because of first} \\ \text{term, } x \neq -2; \\ \text{because of second} \\ \text{term, } x \neq -1. \end{array} \right. \\
 &= \frac{x+1}{x+2} + \frac{x+2}{x+1} \\
 &= \frac{(x+1)^2 + (x+2)^2}{(x+1)(x+2)} && \therefore \text{domain is} \\
 &= \frac{2x^2 + 6x + 5}{x^2 + 3x + 2} && (-\infty, -2) \cup (-2, -1) \cup (-1, \infty).
 \end{aligned}$$

(2)[5 points] Evaluate

$$\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$$

$$= \lim_{x \rightarrow 2} \frac{(x-2)(x+3)}{(x-2)}$$

$$= 5$$

(3)[5 points] Evaluate

$$\lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x - 7}$$

$$= \lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x - 7} \cdot \frac{\sqrt{x+2} + 3}{\sqrt{x+2} + 3}$$

$$= \lim_{x \rightarrow 7} \frac{x+2 - 9}{(x-7)(\sqrt{x+2} + 3)}$$

$$= \lim_{x \rightarrow 7} \frac{(x-7)}{(x-7)(\sqrt{x+2} + 3)}$$

$$= \frac{1}{6}$$