2.3 #26: $(2,2)$, $(-2,-2)$

$$m = \frac{2-(-2)}{2-(-2)} = \frac{4}{4} = 1$$

$$\therefore\ y - y_0 = m(x - x_0)$$

$$y - 2 = 1(x-2)$$

$$y = x$$

2.3 #34: $(0,-2)$, perpendicular to $3x+4y+5=0$

$$3x+4y+5=0$$

$$4y = -3x-5$$

$$y = \left(-\tfrac{3}{4}\right)x - \tfrac{5}{4} \Big\}\ \text{slope } -\tfrac{3}{4}.$$

$\therefore$ desired line has slope $m = \dfrac{-1}{\left(-\frac{3}{4}\right)} = \dfrac{4}{3}$.

$\therefore$ equation is $y-(-2) = \left(\tfrac{4}{3}\right)(x-0)$

$$y+2 = \tfrac{4}{3}x$$

$$y = \tfrac{4}{3}x - 2$$

2.3 #44: $f(x) = 2x+5$, $g(x) = \frac{3}{2}x+5$

$$2x+5 = \tfrac{3}{2}x+5$$

$$\left(2-\tfrac{3}{2}\right)x = 5-5=0$$

$$\therefore\ \tfrac{1}{2}x = 0$$

$$x = 0$$

$$f(0) = 2(0)+5 = 5$$

$\therefore$ point of intersection is $(0,5)$

2.3 #48: $f(x) = \frac{4}{3}x - 5$

$$\frac{f(x+h)-f(x)}{h}$$

$$= \frac{\left[\left(\frac{4}{3}\right)(x+h)-5\right] - \left[\frac{4}{3}x-5\right]}{h}$$

$$= \frac{\cancel{\left(\tfrac{4}{3}\right)x} + \left(\tfrac{4}{3}\right)h \cancel{-5} \cancel{-\left(\tfrac{4}{3}\right)x} \cancel{+5}}{h}$$

$$= \frac{\left(\frac{4}{3}\right)\cancel{h}}{\cancel{h}}$$

$$= \frac{4}{3}.$$

2.4 #18: $f(x) = -x^2+6x-9$

(a) x-intercepts:

$$-x^2+6x-9=0$$
$$x^2-6x+9=0$$
$$(x-3)(x-3)=0$$
$$\therefore x=3$$
$$\therefore (3,0)$$

y-intercept:

$$y=-0^2+6\cdot0-9$$
$$y=-9$$
$$\therefore (0,-9)$$

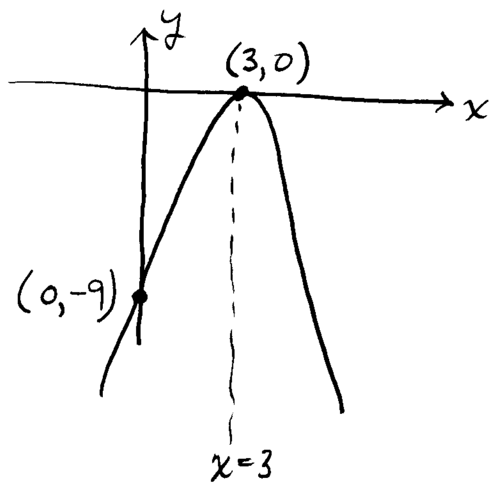
(b.)

$$f(x) = -[x^2-6x+9]$$
$$= -[(x-3)^2-9+9]$$
$$= -(x-3)^2$$

(c) vertex : $(3,0)$

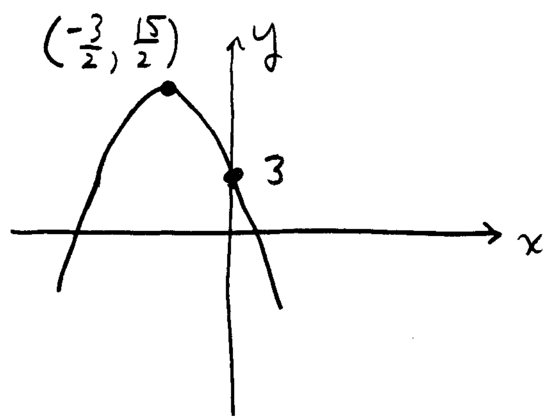
axis of symmetry: $x=3$

(d.)



2.4 #20:

$$f(x) = -2x^2-6x+3$$
$$= -2\left[x^2+3x-\tfrac{3}{2}\right]$$
$$= -2\left[\left(x+\tfrac{3}{2}\right)^2-\tfrac{9}{4}-\tfrac{3}{2}\right]$$
$$= -2\left[\left(x+\tfrac{3}{2}\right)^2-\tfrac{15}{4}\right]$$
$$= -2\left(x+\tfrac{3}{2}\right)^2+\tfrac{15}{2}$$



$\therefore$ maximum of f is $\frac{15}{2}$, range is $(-\infty, \frac{15}{2}]$.

2.4 #44: $y = x^2 - 6x + 1$, $y = -x^2 + 2x + 1$

$= (x-3)^2 - 9 + 1$ $\qquad = -(x^2 - 2x - 1)$

$= (x-3)^2 - 8$ $\qquad = -[(x-1)^2 - 1 - 1]$

$\qquad = -(x-1)^2 + 2$

Points of intersection: $x^2 - 6x + 1 = -x^2 + 2x + 1$

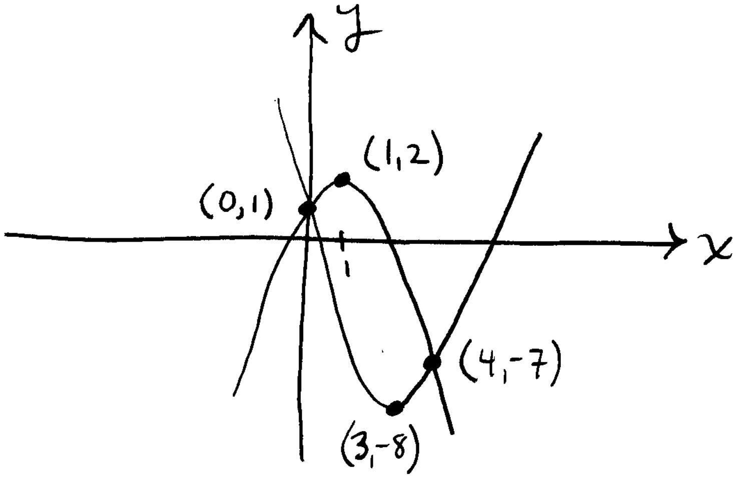
$2x^2 - 8x = 0$

$2x(x-4) = 0$

$x = 0, \quad x = 4$

$\therefore y = 1, \quad y = -7$ } using $y = x^2 - 6x + 1$

$\therefore (0,1), \quad (4,-7)$



2.5 #28: $y = |-x^2 - 4x + 5|$

x-intercepts: $|-x^2 - 4x + 5| = 0$

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x = -5, \ x = 1$

$\therefore (-5,0), (1,0)$

y-intercept: $y = |-0^2 - 4 \cdot 0 + 5|$

$y = 5$

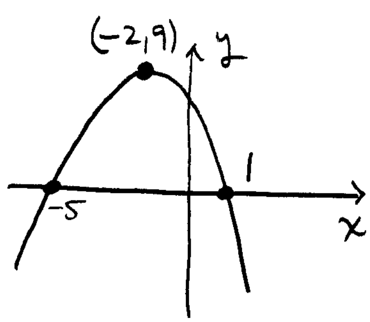
$\therefore (0,5)$

$y = -x^2 - 4x + 5$

$= -[x^2 + 4x - 5]$

$= -[(x+2)^2 - 4 - 5]$

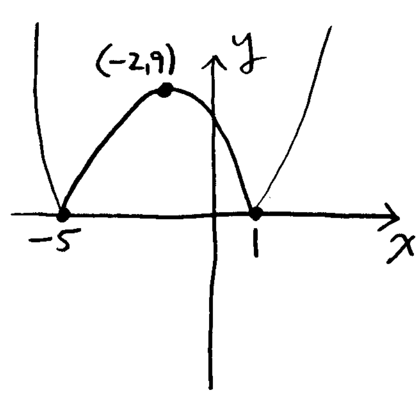
$= -(x+2)^2 + 9$



$\therefore$ graph of

$y = |-x^2 - 4x + 5|$

$= |-(x+2)^2 + 9|$ is

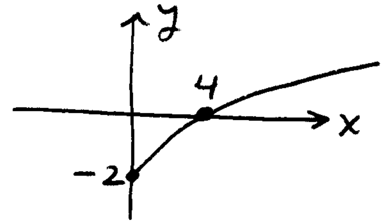


2.5 #30: $y = |\sqrt{x} - 2|$

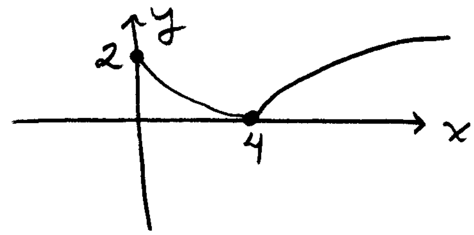
x-intercepts: $|\sqrt{x} - 2| = 0$
$$\sqrt{x} - 2 = 0$$
$$\sqrt{x} = 2$$
$$x = 4$$
$\therefore (4, 0)$

y-intercept: $y = |\sqrt{0} - 2| = 2$
$\therefore (0, 2)$

$y = \sqrt{x} - 2$ has graph



$\therefore y = |\sqrt{x} - 2|$ has graph



2.6 #14: $f(x) = \dfrac{x+1}{x}$, $g(x) = \dfrac{1}{x}$.

$$(f \circ g)(x) = f(g(x)) = \frac{\left(\frac{1}{x}\right) + 1}{\left(\frac{1}{x}\right)} = \frac{\frac{1+x}{x}}{\frac{1}{x}} = \left(\frac{1+x}{x}\right)\left(\frac{x}{1}\right) = 1 + x$$

$\hookrightarrow$ domain of $f \circ g$ is $(-\infty, 0) \cup (0, \infty)$.

$$(g \circ f)(x) = g(f(x)) = \frac{1}{\left(\frac{x+1}{x}\right)} = \frac{x}{x+1}$$

$\hookrightarrow$ requires $x \neq 0$ and also $x + 1 \neq 0$,
i.e. $x \neq -1$

$\therefore$ domain of $g \circ f$ is $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$

2.6 #20: $f(x) = x^3 - 4$; $g(x) = \sqrt[3]{x+3}$

$$(f \circ g)(x) = f(g(x))$$
$$= \left(\sqrt[3]{x+3}\right)^3 - 4$$
$$= x + 3 - 4$$
$$= x - 1$$

$$(g \circ f)(x) = g(f(x))$$
$$= \sqrt[3]{(x^3 - 4) + 3}$$
$$= \sqrt[3]{x^3 - 1}$$

2.6 # 24: $f(x) = \dfrac{x+4}{x}$

$$(f \circ f)(x) = f(f(x)) = \frac{\left(\frac{x+4}{x}\right)+4}{\left(\frac{x+4}{x}\right)} = \frac{\left(\frac{x+4+4x}{x}\right)}{\left(\frac{x+4}{x}\right)} = \left(\frac{5x+4}{x}\right)\left(\frac{x}{x+4}\right) = \frac{5x+4}{x+4}$$

$$\left(f \circ \left(\frac{1}{f}\right)\right)(x) = f\left(\frac{1}{\left(\frac{x+4}{x}\right)}\right) = f\left(\frac{x}{x+4}\right) = \frac{\left(\frac{x}{x+4}\right)+4}{\frac{x}{x+4}}$$

$$= \frac{\left(\frac{x+4x+16}{x+4}\right)}{\left(\frac{x}{x+4}\right)}$$

$$= \left(\frac{5x+16}{x+4}\right)\left(\frac{x+4}{x}\right)$$

$$= \frac{5x+16}{x}$$

2.6 # 34: $F(x) = 1 + |2x+9|$

Let $g(x) = 2x+9$

Then $f(x) = 1 + |x|$

Check: $(f \circ g)(x) = f(g(x)) = 1 + |2x+9| = F(x)$ ✓

2.7 # 22: $f(x) = 2 + \dfrac{3}{\sqrt{x}}$, $\quad x>0$, $y>2$

$y = 2 + \dfrac{3}{\sqrt{x}}$

$x = 2 + \dfrac{3}{\sqrt{y}}$

$x-2 = \dfrac{3}{\sqrt{y}}$

$\sqrt{y} = \dfrac{3}{x-2}$

$y = \left(\dfrac{3}{x-2}\right)^2$

$\therefore f^{-1}(x) = \left(\dfrac{3}{x-2}\right)^2$

Domain of f^{-1} = Range of f which is $(2, \infty)$

Range of f^{-1} = Domain of f which is $(0, \infty)$.

2.7 #26: $f(x) = 1 - x^3$

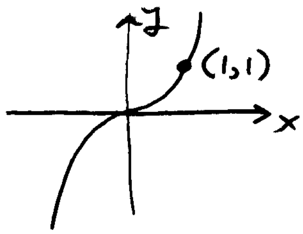
$$y = 1 - x^3$$

$$x = 1 - y^3$$

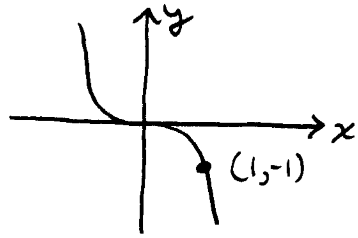
$$y^3 = 1 - x$$

$$y = \sqrt[3]{1-x}$$

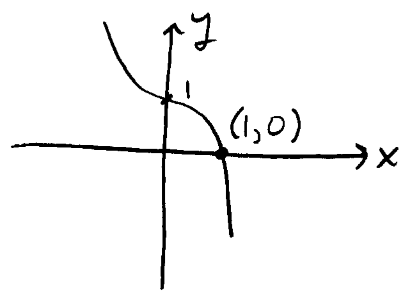
① $y = x^3$

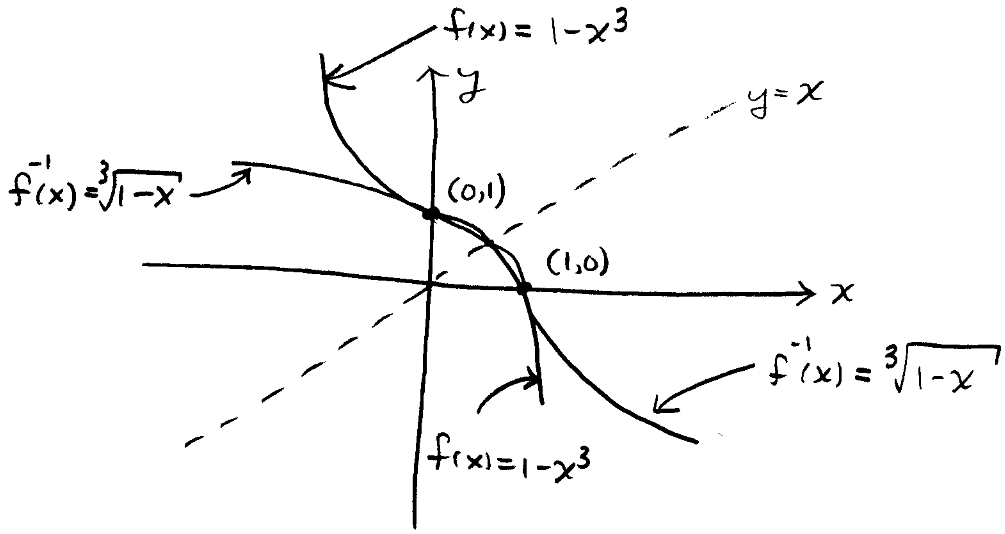


② $y = -x^3$

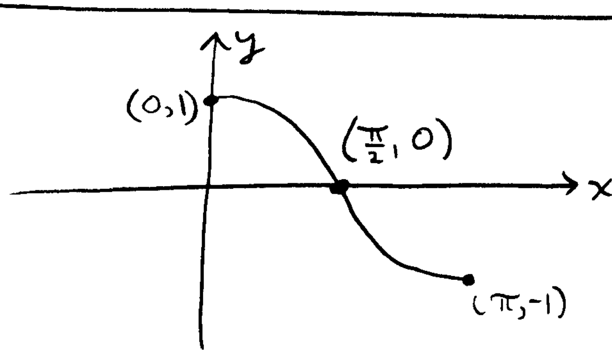


③ $y = 1 - x^3$

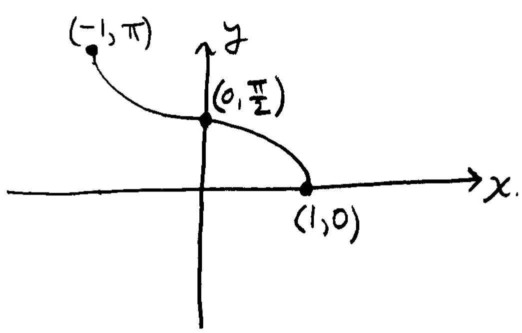




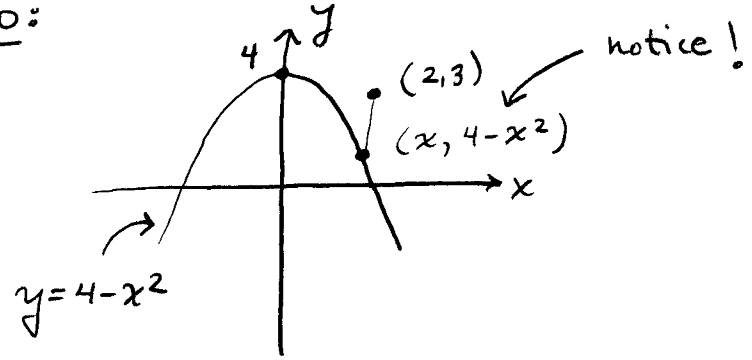
2.7 #38: $y = f(x)$:



$\therefore \quad y = f^{-1}(x)$:



2.8 #10:

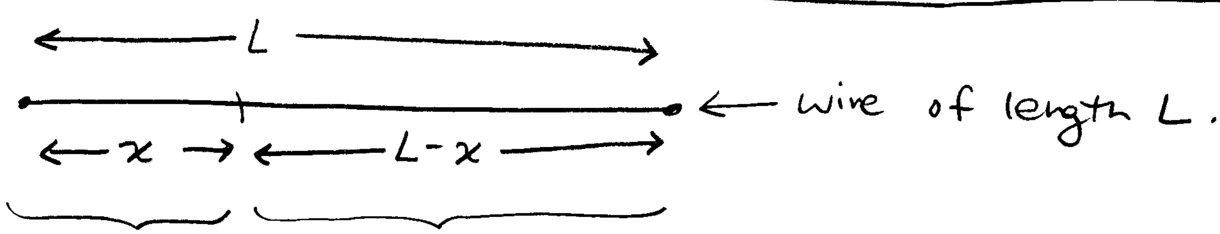


Let $f(x)$ = distance from $(2,3)$ to $(x, 4-x^2)$.

$$f(x) = \sqrt{(x-2)^2 + (4-x^2-3)^2}$$

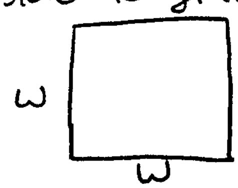
$$= \sqrt{(x-2)^2 + (1-x^2)^2}, \quad \text{domain } (-\infty, \infty).$$

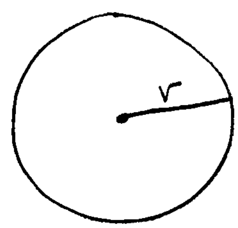
2.8 #18:



Construct circle of radius r:

Construct square of side length w:





Perimeter of square must be x,

$\therefore 4w = x$

$\therefore w = \dfrac{x}{4}$

$\therefore$ Area of square $= w^2 = \left(\dfrac{x}{4}\right)^2$

Circumference must be $L-x$

$\therefore 2\pi r = L-x$

$\therefore r = \dfrac{L-x}{2\pi}$

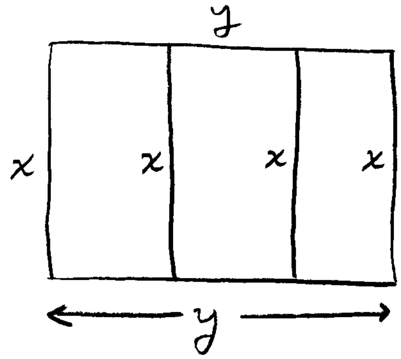
$\therefore$ Area of circle $= \pi r^2$

$= \pi\left(\dfrac{L-x}{2\pi}\right)^2$

Letting $A(x)$ = total area of both figures,

$$A(x) = \left(\frac{x}{4}\right)^2 + \pi\left(\frac{L-x}{2\pi}\right)^2, \quad 0 \le x \le L$$

2.8 #30:



$4x + 2y = 8000$ m } constraint

Let A = total area.

$A = xy$.

Using $4x + 2y = 8000$, $y = \dfrac{8000 - 4x}{2} = 4000 - 2x$

$\therefore A(x) = x(4000 - 2x)$, $0 \leq x \leq 2000$

(or < here).

$\therefore$ Find x which maximizes $A(x) = x(4000 - 2x)$ on $[0, 2000]$.

Question did not ask us to find solution, but rather just the objective function $A(x)$. It is not hard to find the maximum of $A(x)$ here though:

$$A(x) = 4000x - 2x^2 = -2\left[x^2 - 2000x\right]$$

$$= -2\left[(x - 1000)^2 - 1{,}000{,}000\right]$$

$$= -2(x - 1000)^2 + 2{,}000{,}000$$

$\therefore$ Area is a maximum of 2,000,000 m^2 when $x = 1000$ m.

The corresponding value for $y = \dfrac{8000 - 4x}{2}$

$$= \frac{8000 - 4(1000)}{2}$$

$$= 2000 \text{ m}.$$

$\therefore$ the land should have dimensions 1000 m × 2000 m.