1.1 #34:

$$2 \le \frac{4x+2}{-3} \le 10$$

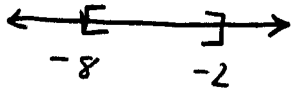
$$-6 \ge 4x+2 \ge -30$$

$$-30 \le 4x+2 \le -6$$

$$-32 \le 4x \le -8$$

$$-8 \le x \le -2$$

$\therefore [-8,-2]$,



1.1 #54:

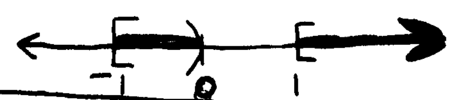
$$\frac{(1+x)(1-x)}{x} \le 0$$

$1+x=0$, $1-x=0$, $x=0$

$x=-1$, $x=1$, $x=0$

	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
$1+x$:	$-$		$+$		$+$		$+$
$1-x$:	$+$		$+$		$+$		$-$
x :	$-$		$-$		$+$		$+$
$\frac{(1+x)(1-x)}{x}$:	$+$	0	$-$	undef.	$+$	0	$-$

$$\therefore \frac{(1+x)(1-x)}{x} \le 0 \text{ on } [-1,0)\cup[1,\infty)$$



1.2 #30: $|5v-4|=7$

$5v-4=7$ or $5v-4=-7$

$v=\frac{7+4}{5}$ $\quad$ $v=\frac{-7+4}{5}$

$v=\frac{11}{5}$, $\quad$ $v=\frac{-3}{5}$

1.2 #42: $|6x+4|>4$

$6x+4>4$, $\quad$ $6x+4<-4$

$6x>0$ $\quad$ $6x<-8$

$x>0$ $\quad$ $x<-\frac{8}{6}$

$x<-\frac{4}{3}$

$\therefore (-\infty,-\frac{4}{3})\cup(0,\infty)$

1.3 #32: $A(-\frac{5}{3},4)$, $B(-\frac{2}{3},-1)$

$$d(A,B)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{\left(-\frac{2}{3}+\frac{5}{3}\right)^2+(-1-4)^2}$$

$$=\sqrt{1+25}$$

$$=\sqrt{26}.$$

1.4 #12: $x^2+y^2+3x-16y+63=0$

$$\left(x+\frac{3}{2}\right)^2-\frac{9}{4}+(y-8)^2-64+63=0$$

$$\left(x+\frac{3}{2}\right)^2+(y-8)^2=64-63+\frac{9}{4}$$

$$\left(x+\frac{3}{2}\right)^2+(y-8)^2=\frac{13}{4}$$

$\therefore$ center $(-\frac{3}{2},8)$, radius $\sqrt{\frac{13}{4}}$.

1.4 #20: diameter with endpoints $(4,2)$, $(-3,5)$

$\therefore$ centre is $\left(\frac{4+(-3)}{2}, \frac{2+5}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right)$

radius is $r = \sqrt{\left(4-\frac{1}{2}\right)^2 + \left(2-\frac{7}{2}\right)^2} = \sqrt{\frac{49}{4} + \frac{9}{4}} = \sqrt{\frac{29}{2}}$

$\therefore$ equation is $\left(x-\frac{1}{2}\right)^2 + \left(y-\frac{7}{2}\right)^2 = \frac{29}{2}$

1.4 #56: $y = \frac{(x+2)(x-8)}{x+1}$

x-intercepts: $0 = \frac{(x+2)(x-8)}{x+1}$

$\therefore x = -2, 8$

$\therefore (-2,0), (8,0)$

y-intercepts: $y = \frac{(0+2)(0-8)}{0+1}$

$= -16$

$\therefore (0,-16)$

Symmetry:

x-axis: $y = \frac{(x+2)(x-8)}{x+1}$

$y \leftrightarrow -y$

$-y = \frac{(x+2)(x-8)}{x+1}$

not equivalent, so no x-axis symmetry.

y-axis: $y = \frac{(x+2)(x-8)}{x+1}$

$x \leftrightarrow -x$:

$y = \frac{(-x+2)(-x+8)}{-x+1}$

not equivalent, so no y-axis symmetry.

origin: $y = \frac{(x+2)(x-8)}{x+1}$

$(x,y) \leftrightarrow (-x,-y)$:

$-y = \frac{(-x+2)(-x+8)}{-x+1}$

not equivalent, so no origin symmetry.

2.1 #25: $f(x) = \sqrt{\frac{3-x}{x+2}}$ $\Big\}$ require $\underbrace{\frac{3-x}{x+2} \geq 0}$, $x \neq -2$

$3-x=0$, $x+2=0$

$x=3$, $x=-2$

	-3	-2	0	3	4
$3-x$:	$+$		$+$		$-$
$x+2$:	$-$		$+$		$+$
$\frac{(3-x)}{x+2}$:	$-$	un-def	$+$	0	$-$

$\therefore \frac{3-x}{x+2} \geq 0$ on $(-2, 3]$

$\therefore$ domain of f is $(-2, 3]$

2.1 #40: $f(x) = x^3 - x^2 - 2x = 0$

$x(x^2 - x - 2) = 0$

$x(x-2)(x+1) = 0$

$x=0$, $x=2$, $x=-1$

2.1 #50: $f(x) = \frac{1}{2}\sqrt{x^2 - 2x - 3}$

x-intercepts:

$\frac{1}{2}\sqrt{x^2-2x-3} = 0$

$x^2-2x-3 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$

$\therefore (3,0), (-1,0)$

y-intercept:

$f(0) = \frac{1}{2}\sqrt{0^2 - 2(0) - 3}$

$= \underbrace{\frac{1}{2}\sqrt{-3}}_{\text{not a real number.}}$

$\therefore$ no y-intercept.

2.2 #6: $f(x) = \frac{x}{x^2+1}$: even? is $f(-x) = f(x)$?

$f(-x) = \frac{-x}{(-x)^2+1} = \frac{-x}{x^2+1} \neq f(x)$

$\therefore f$ is not even.

odd? is $f(-x) = -f(x)$?

$f(-x) = \frac{-x}{(-x)^2+1} = \frac{-x}{x^2+1} = -f(x)$.

$\therefore f$ is even.

2.2 #42: $f(x) = \frac{1}{x}$; (i) reflected in y-axis : $f(x) = \frac{-1}{x}$,

(ii) shift left 5 units : $f(x) = \frac{-1}{x+5}$

(iii) shift down 10 units : $f(x) = -\frac{1}{x+5} - 10$

2.3 #30: $(1,-3)$; $2x-5y+4=0$

$$y = \frac{2}{5}x + \frac{4}{5}$$

$\therefore m = \frac{2}{5}$.

$\therefore$ equation of line is $y-(-3) = \frac{2}{5}(x-1)$

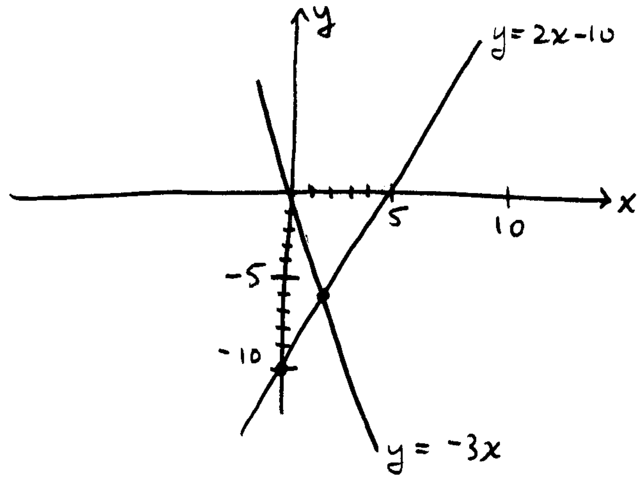
$$y+3 = \frac{2}{5}(x-1) \quad \text{or} \quad y = \frac{2}{5}x - \frac{17}{5}$$

2.3 #46: $f(x) = 2x-10$, $g(x) = -3x$

$$2x-10 = -3x$$
$$5x = 10$$
$$x = 2$$

$\therefore y = f(2) = 2(2)-10 = -6$

$\therefore (2,-6)$



2.4 #14: $f(x) = -x^2+6x-10$

x-intercepts: $-x^2+6x-10=0$

$$x^2-6x+10=0$$

$$x = \frac{6 \pm \sqrt{36-4(1)(10)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{-4}}{2} \leftarrow \text{no solutions, so no x-intercepts}$$

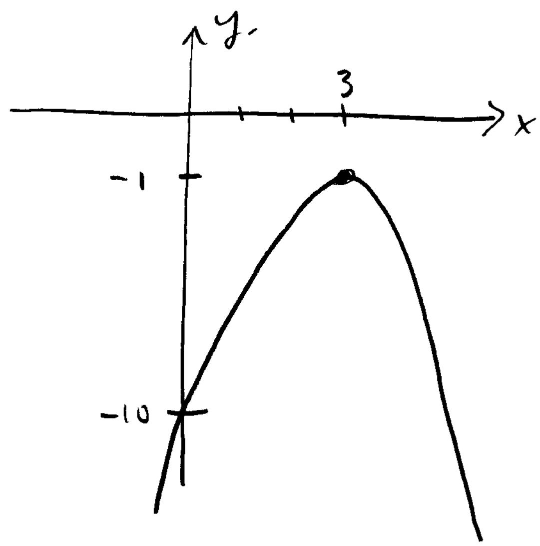
y-intercept: $f(0) = -10$

$\therefore (0,-10)$

$$f(x) = -\left[x^2-6x+10\right]$$
$$= -\left[(x-3)^2-9+10\right]$$
$$= -(x-3)^2-1$$

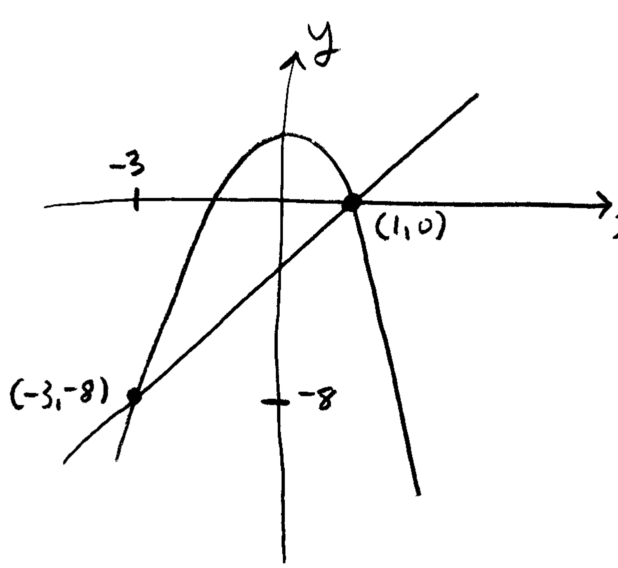
$\therefore$ vertex is $(3,-1)$

axis of symmetry $x=3$



2.4 #42: $y = 2x-2, \quad y = 1-x^2$

$$2x-2 = 1-x^2$$
$$x^2+2x-3=0$$
$$(x+3)(x-1)=0$$
$$x=-3, \quad x=1$$
$$\therefore \; y=2(-3)-2 = -8, \quad y=2(1)-2 = 0$$
$$\therefore (-3,-8), \quad (1,0)$$



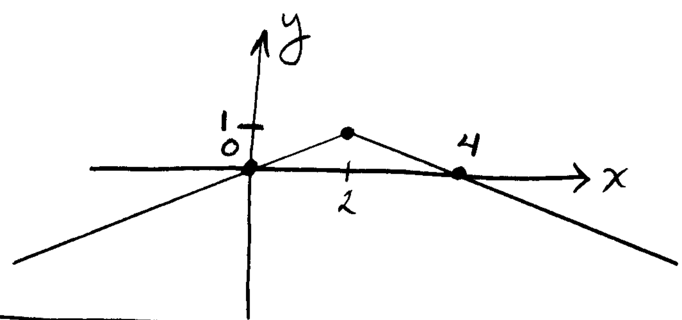
2.5 #22: $y = 1 - \frac{1}{2}|x-2|$

x-intercepts:

$$0 = 1 - \tfrac{1}{2}|x-2|$$
$$\tfrac{1}{2}|x-2| = 1$$
$$|x-2| = 2$$
$$x-2=2, \quad x-2=-2$$
$$x=4, \quad x=0$$
$$\therefore (4,0), \quad (0,0)$$

y-intercepts:

$$y = 1-\tfrac{1}{2}|0-2|$$
$$= 1-\tfrac{1}{2}(2)$$
$$= 0$$
$$\therefore (0,0)$$



2.5 #29: $y = \big|\,|x|-2\,\big|$:

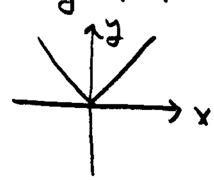
x-int:

$$0 = \big|\,|x|-2\,\big|$$
$$\therefore 0 = |x|-2$$
$$|x| = 2$$
$$x = 2, -2,$$
$$\therefore (2,0), \quad (-2,0)$$

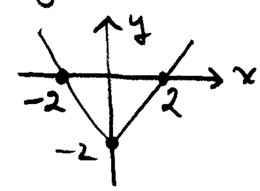
y-int: $y = \big|\,|0|-2\,\big| = 2$

$$\therefore (0,2)$$

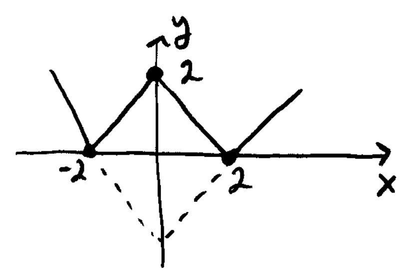
① $y = |x|$



② $y = |x|-2$



③ $y = \big|\,|x|-2\,\big|$:



2.6 #12:

$f(x) = x^2 - x + 5$

$g(x) = -x + 4$

$(f \circ g)(x) = f(g(x)) = \underbrace{(-x+4)^2 - (-x+4) + 5}_{\text{domain: } (-\infty, \infty)} = x^2 - 8x + 16 + x - 4 + 5$

$= x^2 - 7x + 17$

$(g \circ f)(x) = g(f(x)) = \underbrace{-(x^2 - x + 5) + 4}_{\text{domain: } (-\infty, \infty)} = -x^2 + x - 1$

2.6 #32: $F(x) = \sqrt{9x^2 + 16}$: let $g(x) = 9x^2 + 16$, $f(x) = \sqrt{x}$.

2.7 #28:

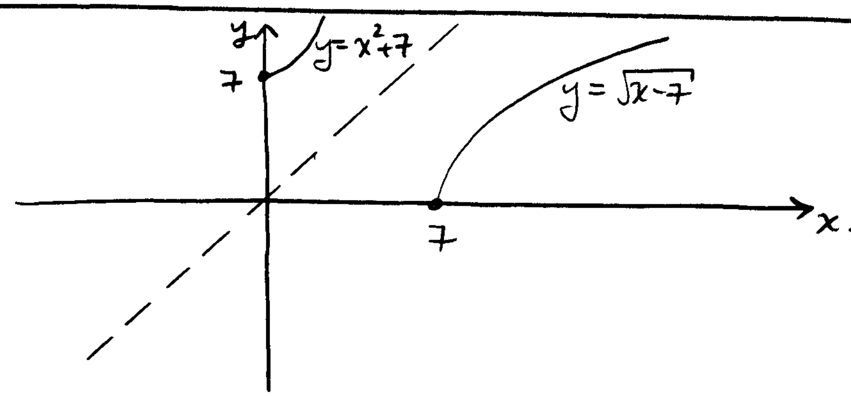
$f(x) = \sqrt{x-7}$

$y = \sqrt{x-7}$

$x \leftrightarrow y$: $x = \sqrt{y-7}$

$x^2 = y - 7$

$y = x^2 + 7$



2.7 #32:

$f(x) = \dfrac{1-x}{x-2}$ } domain $(-\infty, 2) \cup (2, \infty)$

$\therefore$ range $(-\infty, -1) \cup (-1, \infty)$

$y = \dfrac{1-x}{x-2}$

$x = \dfrac{1-y}{y-2}$

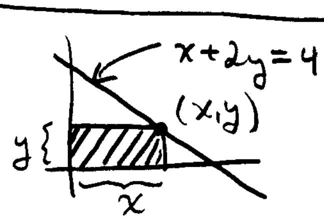
$xy - 2x = 1 - y$

$y(x+1) = 1 + 2x$

$\therefore\ y = f^{-1}(x) = \dfrac{1+2x}{1+x}$ } domain $(-\infty, -1) \cup (-1, \infty)$

$\therefore$ range $(-\infty, 2) \cup (2, \infty)$

2.8 #7:



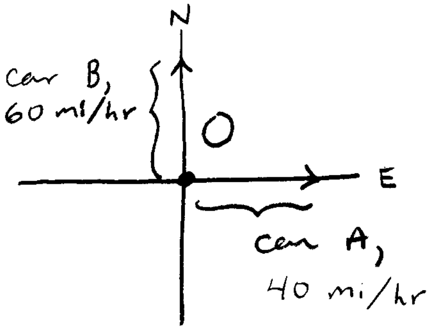
Let A = area.

$A = xy$

since $x + 2y = 4$, $y = \dfrac{4-x}{2}$.

$\therefore\ A(x) = x\left(\dfrac{4-x}{2}\right)$

2.8 #23:



$t =$ time since car B passed O.

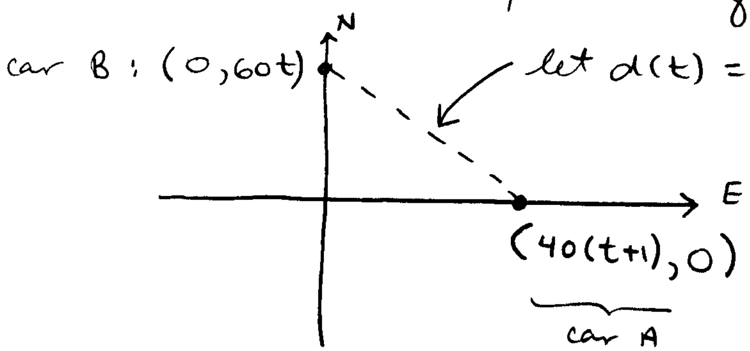
$\therefore$ distance between car A and O at time $t = (t+1)\text{ hrs} \cdot 40\,\frac{\text{mi}}{\text{hr}}$

$= 40(t+1)$ miles.

distance between car B and O at time $t = t\text{ hrs} \cdot 60\,\text{mi/hr}$

$= 60t$ miles

$\therefore$ At time t positions of cars are:

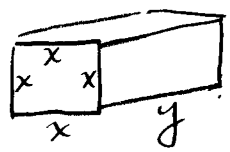


let $d(t)$ = distance at time t.

$$\therefore\ d(t) = \sqrt{[40(t+1)-0]^2 + [60t-0]^2}$$

$$= \sqrt{1600(t+1)^2 + 3600t^2}, \quad t \geq 0$$

2.8 #26:

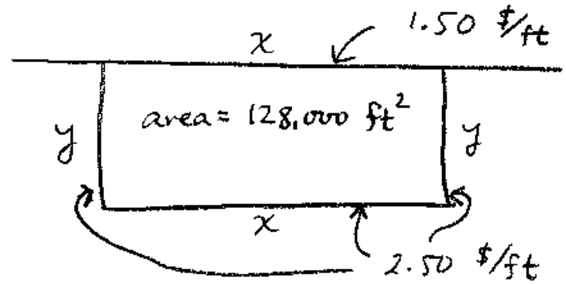


$V = x^2 y$ where $4x + y = 108$

$\therefore\ y = 108 - 4x$

$$\therefore\ V(x) = x^2(108-4x), \quad 0 \leq x \leq 27$$

2.8 #31:



$xy = 128{,}000$

$\therefore y = \dfrac{128{,}000}{x}$.

Let C = cost of construction.

$$C = (x \text{ ft})(1.50\ \$/\text{ft}) + (x + 2y \text{ ft})(2.50\ \$/\text{ft})$$

$$= 1.5x + 2.5x + 2(2.50)y$$

$$= 4x + 5y$$

$$\therefore C(x) = 4x + 5\left(\frac{128{,}000}{x}\right), \qquad 0 < x < \infty$$

3.2 #10:

$$
\begin{array}{r}
5x^4 - 6x^3 + 21x^2 - 27x + 51 \\
x^2 + x - 1 \,\big)\, 5x^6 - x^5 + 10x^4 + 0x^3 + 3x^2 - 2x + 4 \\
-(5x^6 + 5x^5 - 5x^4) \qquad\qquad\qquad\qquad \\
\hline
-6x^5 + 15x^4 + 0x^3 + 3x^2 - 2x + 4 \\
-(-6x^5 - 6x^4 + 6x^3) \qquad\qquad\qquad \\
\hline
21x^4 - 6x^3 + 3x^2 - 2x + 4 \\
-(21x^4 + 21x^3 - 21x^2) \qquad\qquad \\
\hline
-27x^3 + 24x^2 - 2x + 4 \\
-(-27x^3 - 27x^2 + 27x) \qquad \\
\hline
51x^2 - 29x + 4 \\
-(51x^2 + 51x - 51) \\
\hline
-80x + 55
\end{array}
$$

$$\therefore 5x^6 - x^5 + 10x^4 + 3x^2 - 2x + 4 = (x^2 + x - 1)(5x^4 - 6x^3 + 21x^2 - 27x + 51) + (-80x + 55)$$

3.2 #.22 : $f(x) = 14x^4 - 60x^3 + 49x^2 - 21x + 19$

$f(1) = ?$

$$\begin{array}{c|ccccc} 1 & 14 & -60 & 49 & -21 & 19 \\ & & 14 & -46 & 3 & -18 \\ \hline & 14 & -46 & 3 & -18 & \boxed{1} \end{array} \longleftarrow \therefore f(1) = 1.$$

3.4 #25: $f(x) = x^5 + 4x^4 - 6x^3 - 24x^2 + 5x + 20$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20.$

$f(1)$:

$$\left.\begin{array}{c|cccccc} 1 & 1 & 4 & -6 & -24 & 5 & 20 \\ & & 1 & 5 & -1 & -25 & -20 \\ \hline & 1 & 5 & -1 & -25 & -20 & \boxed{0} \end{array}\right\} \therefore (x-1) \text{ is a factor,}$$

$$\therefore f(x) = (x-1)(x^4 + 5x^3 - x^2 - 25x - 20)$$

possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

$$\begin{array}{c|ccccc} 1 & 1 & 5 & -1 & -25 & -20 \\ & & 1 & 6 & 5 & -20 \\ \hline & 1 & 6 & 5 & -20 & -40 \end{array} \times$$

$$\left.\begin{array}{c|ccccc} -1 & 1 & 5 & -1 & -25 & -20 \\ & & -1 & -4 & 5 & 20 \\ \hline & 1 & 4 & -5 & -20 & \boxed{0} \end{array}\right\} \therefore (x+1) \text{ is a factor.}$$

$$\therefore f(x) = (x-1)(x+1)(x^3 + 4x^2 - 5x - 20)$$

possible rational zeros $-1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

$$\begin{array}{c|cccc} -1 & 1 & 4 & -5 & -20 \\ & & -1 & -3 & 8 \\ \hline & 1 & 3 & -8 & \boxed{-12} \end{array} \times$$

$$\begin{array}{c|cccc} 2 & 1 & 4 & -5 & -20 \\ & & 2 & 12 & 14 \\ \hline & 1 & 6 & 7 & \boxed{-6} \end{array} \times$$

$$\begin{array}{c|cccc} -2 & 1 & 4 & -5 & -20 \\ & & -2 & -4 & 18 \\ \hline & 1 & 2 & -9 & \boxed{-2} \end{array} \times$$

$$\begin{array}{c|cccc} 4 & 1 & 4 & -5 & -20 \\ & & 4 & 32 & 108 \\ \hline & 1 & 8 & 27 & \boxed{88} \end{array} \times$$

$$\left.\begin{array}{c|cccc} -4 & 1 & 4 & -5 & -20 \\ & & -4 & 0 & 20 \\ \hline & 1 & 0 & -5 & \boxed{0} \end{array}\right\} \therefore x+4 \text{ is a factor}$$

$$\therefore f(x) = (x-1)(x+1)(x+4)(x^2-5) = (x-1)(x+1)(x+4)(x-\sqrt{5})(x+\sqrt{5})$$

and zeros are $x = 1, -1, -4, \sqrt{5}, -\sqrt{5}.$

3.4 #29:

$f(x) = \underbrace{16}_{}x^5 - 24x^4 + 25x^3 + 39x^2 - 23x + 3$ (10)

possible s: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$.

possible p: $\pm 1, \pm 3$

$$\begin{array}{r|rrrrrr} 1 & 16 & -24 & 25 & 39 & -23 & 3 \\ & & 16 & -8 & 17 & 56 & 33 \\ \hline & 16 & -8 & 17 & 56 & 33 & \boxed{36} \end{array} \quad \times$$

$$\begin{array}{r|rrrrrr} -1 & 16 & -24 & 25 & 39 & -23 & 3 \\ & & -16 & 40 & -65 & 26 & -3 \\ \hline & 16 & -40 & 65 & -26 & 3 & \boxed{0} \end{array} \Bigg\} \therefore \ x+1 \text{ is a factor.}$$

$\therefore f(x) = (x+1)(16x^4 - 40x^3 + 65x^2 - 26x + 3)$

$$\begin{array}{r|rrrrr} \frac{1}{4} & 16 & -40 & 65 & -26 & 3 \\ & & 4 & -9 & 14 & -3 \\ \hline & 16 & -36 & 56 & -12 & \boxed{0} \end{array} \leftarrow \therefore (x - \tfrac{1}{4}) \text{ is a factor}$$

$\therefore f(x) = (x+1)(x-\frac{1}{4})(16x^3 - 36x^2 + 56x - 12)$

$$\begin{array}{r|rrrr} \frac{1}{4} & 16 & -36 & 56 & -12 \\ & & 4 & -8 & 12 \\ \hline & 16 & -32 & 48 & \boxed{0} \end{array} \Bigg\} (x - \tfrac{1}{4}) \text{ is again a factor}$$

$\therefore f(x) = (x+1)(x-\frac{1}{4})(x-\frac{1}{4})(16x^2 - 32x + 48)$

$= 16(x+1)(x-\frac{1}{4})(x-\frac{1}{4})\underbrace{(x^2 - 2x + 3)}_{b^2 - 4ac < 0}$, so no more zeros.

zeros of f are $x = -1, \frac{1}{4}$.