Question 1:

(a)[**4 points**] Express as a single simplified fraction:

$$\frac{x}{x+1}+\frac{1}{x-1}$$

$$= \frac{x(x-1) + x+1}{(x+1)(x-1)}$$

$$= \frac{x^2 - x + x + 1}{x^2-1}$$

$$= \frac{x^2+1}{x^2-1}$$

(b)[**3 points**] Expand and simplify:

$$2\left(\frac{a+b}{2}\right)^2 - \left(\frac{a^2+b^2}{2}\right)$$

$$= \cancel{2}\ \frac{a^2+2ab+b^2}{\cancel{4}\ 2} - \frac{a^2+b^2}{2}$$

$$= \frac{\cancel{a^2} + 2ab \cancel{+b^2} - \cancel{a^2} \cancel{-b^2}}{2}$$

$$= ab$$

(c)[**3 points**] Simplify using only positive exponents:

$$\left(\frac{(2x)^3y^{-3}}{x^{-3}(2y)^{-2}}\right)$$

$$= \frac{2^3x^3y^{-3}}{x^{-3}2^{-2}y^{-2}}$$

$$= \frac{2^5x^6}{y}$$

Question 2:

(a)[5 points] Solve and state your answer using interval notation

$$-3 \leq \frac{2}{3} - 5x \leq 2$$

$$\frac{-3}{1} - \frac{2}{3} \leq -5x \leq \frac{2}{1} - \frac{2}{3}$$

$$\frac{-9-2}{3} \leq -5x \leq \frac{6-2}{3}$$

$$-\frac{11}{3} \leq -5x \leq \frac{4}{3}$$

$$\frac{11}{15} \geq x \geq -\frac{4}{15}$$

$$-\frac{4}{15} \leq x \leq \frac{11}{15}$$

$$\left[-\frac{4}{15}, \frac{11}{15}\right]$$

(b)[5 points] Solve and state your answer using interval notation

$$x^2 + x > 12$$

$$x^2 + x - 12 > 0$$

$$(x-3)(x+4) > 0$$

$(x-3)(x+4) = 0$ at $x = 3$, $x = -4$

$x-3$:	$-$		$-$		$+$
$x+4$:	$-$		$+$		$+$
$(x-3)(x+4)$:	$+$	0	$-$	0	$+$
	-5	-4	0	3	4

$\therefore (x-3)(x+4) > 0$ on

$(-\infty, -4) \cup (3, \infty)$

Question 3:

(a)[4 points] Solve for x

$$\left|\frac{5x-3}{7}\right| = \frac{1}{2}$$

$$\frac{5x-3}{7} = \frac{1}{2} \quad \text{or} \quad \frac{5x-3}{7} = \frac{-1}{2}$$

$$5x-3 = \frac{7}{2} \qquad 5x-3 = -\frac{7}{2}$$

$$5x = \frac{7}{2}+3 \qquad 5x = -\frac{7}{2}+3$$

$$5x = \frac{7}{2}+\frac{6}{2} \qquad 5x = -\frac{7}{2}+\frac{6}{2}$$

$$5x = \frac{13}{2} \qquad 5x = \frac{-1}{2}$$

$$x = \frac{13}{10} \qquad x = \frac{-1}{10}$$

$$\therefore\ x = \frac{13}{10}, \frac{-1}{10}$$

(b)[6 points] Solve and state your answer using interval notation

$$\left|\frac{8-11x}{-3}\right| \geq 5$$

$$\frac{8-11x}{-3} \geq 5 \quad \text{or} \quad \frac{8-11x}{-3} \leq -5$$

$$8-11x \leq -15 \qquad 8-11x \geq 15$$

$$-11x \leq -23 \qquad -11x \geq 7$$

$$x \geq \frac{23}{11} \qquad x \leq -\frac{7}{11}$$

$$\therefore\ \left(-\infty, -\frac{7}{11}\right] \cup \left[\frac{23}{11}, \infty\right)$$

Question 4:

(a)[5 points] The distance from $(-2,5)$ to (a,a) is 9 units. Find all possible values of a.

$$9 = \sqrt{(a-(-2))^2 + (a-5)^2}$$

$$81 = (a+2)^2 + (a-5)^2$$

$$81 = a^2 + 4a + 4 + a^2 - 10a + 25$$

$$\therefore\ 2a^2 - 6a - 52 = 0$$

$$a^2 - 3a - 26 = 0$$

$$\therefore\ a = \frac{-(-3) \pm \sqrt{9 - 4(1)(-26)}}{2}$$

$$= \frac{3 \pm \sqrt{113}}{2}$$

(b)[5 points] Find the points of intersection of the graphs of $y = 2x^2 - 3x + 1$ and $3x - y + 9 = 0$.

$$\left.\begin{array}{l} y = 2x^2 - 3x + 1 \\ y = 3x + 9 \end{array}\right\} \quad \therefore\ 2x^2 - 3x + 1 = 3x + 9$$

$$2x^2 - 6x - 8 = 0$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x = 4, \qquad x = -1$$

$$\therefore\ y = 3(4) + 9 = 21 \qquad y = 3(-1) + 9 = 6$$

$$\therefore\ (4,21),\ (-1,6)$$

Question 5:

(a)[4 points] Put the equation of the following circle in standard form and state the centre and radius

$$x^2 + y^2 - 3x + 5y = 11$$

$$x^2 - 3x \quad + y^2 + 5y \quad = 11$$

$$\left(x-\frac{3}{2}\right)^2 - \frac{9}{4} + \left(y+\frac{5}{2}\right)^2 - \frac{25}{4} = 11$$

$$\left(x-\frac{3}{2}\right)^2 + \left(y+\frac{5}{2}\right)^2 = \frac{44}{4} + \frac{9}{4} + \frac{25}{4}$$

$$\left(x-\frac{3}{2}\right)^2 + \left(y+\frac{5}{2}\right)^2 = \frac{78}{4}$$

$$\left(x-\frac{3}{2}\right)^2 + \left(y+\frac{5}{2}\right)^2 = \frac{39}{2} .$$

$\therefore$ centre $\left(\frac{3}{2}, -\frac{5}{2}\right)$, radius $\sqrt{\frac{39}{2}}$.

(b)[3 points] Find the x and y intercepts of the graph of $y = \dfrac{(x^2-1)(x^2+2)}{x^2-3}$.

x intercept:

$$0 = \frac{(x^2-1)(x^2+2)}{x^2-3}$$

$$= \frac{(x-1)(x+1)(x^2+2)}{x^2-3}$$

$\therefore$ $x = 1$, $x = -1$

$\therefore$ $(1,0)$, $(-1,0)$

y-intercept:

$$y = \frac{(0^2-1)(0^2+2)}{0^2-3}$$

$$= \frac{(-1)(2)}{-3}$$

$$= \frac{2}{3}$$

$\therefore$ $\left(0, \frac{2}{3}\right)$.

(c)[3 points] Find the zeros of $f(x) = x\sqrt{2x-7} - 7\sqrt{2x-7}$.

Solve

$$x\sqrt{2x-7} - 7\sqrt{2x-7} = 0$$

$$\sqrt{2x-7}\,(x-7) = 0$$

$$2x - 7 = 0 \quad , \quad x - 7 = 0$$

$$x = \frac{7}{2} \quad , \quad x = 7$$

Question 6:

(a)[3 points] Find the domain of $f(x) = \dfrac{2\sqrt{9-x}}{x}$.

Must have $9 - x \geq 0$, $x \neq 0$

$x \leq 9$, $x \neq 0$

$\therefore \quad (-\infty, 0) \cup (0, 9]$.

(b)[7 points] Factor completely

$$f(x) = x^4 + 2x^3 - 2x^2 - 6x - 3$$

possible rational zeros: $\pm 1, \pm 3$

$$\begin{array}{r|rrrrr} -1 & 1 & 2 & -2 & -6 & -3 \\ & & -1 & -1 & 3 & 3 \\ \hline & 1 & 1 & -3 & -3 & \boxed{0} \end{array} \quad \therefore f(x) = (x+1)(x^3 + x^2 - 3x - 3)$$

possible rational zeros of $x^3 + x^2 - 3x - 3$: $\pm 1, \pm 3$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -3 & -3 \\ & & -1 & 0 & 3 \\ \hline & 1 & 0 & -3 & \boxed{0} \end{array} \quad \therefore f(x) = (x+1)(x+1)(x^2 - 3)$$

$$= (x+1)(x+1)(x - \sqrt{3})(x + \sqrt{3})$$

Question 7:

(a)[5 points] Find the equation of the line through the midpoint of $(-1,-7)$ and $(7,6)$ which is parallel to the line $5x-3y-4=0$.

$5x-3y-4=0$

$3y = 5x-4$

$y = (\frac{5}{3})x - \frac{4}{3}$

$\therefore m = \frac{5}{3}$

midpoint of $(-1,-7)$ & $(7,6)$

is $\left(\frac{-1+7}{2}, \frac{-7+6}{2}\right) = (3, -\frac{1}{2})$

$\therefore$ line through $(3, -\frac{1}{2})$ with $m = \frac{5}{3}$:

$y-(-\frac{1}{2}) = \frac{5}{3}(x-3)$

$y+\frac{1}{2} = \frac{5}{3}x - 5$

or $y = \frac{5}{3}x - \frac{11}{2}$

(b)[5 points] Put the parabola $y=-x^2+6x+7$ in standard form and state the vertex and axis of symmetry.

$y = -x^2+6x+7$

$= -[x^2-6x-7]$

$= -[(x-3)^2-9-7]$

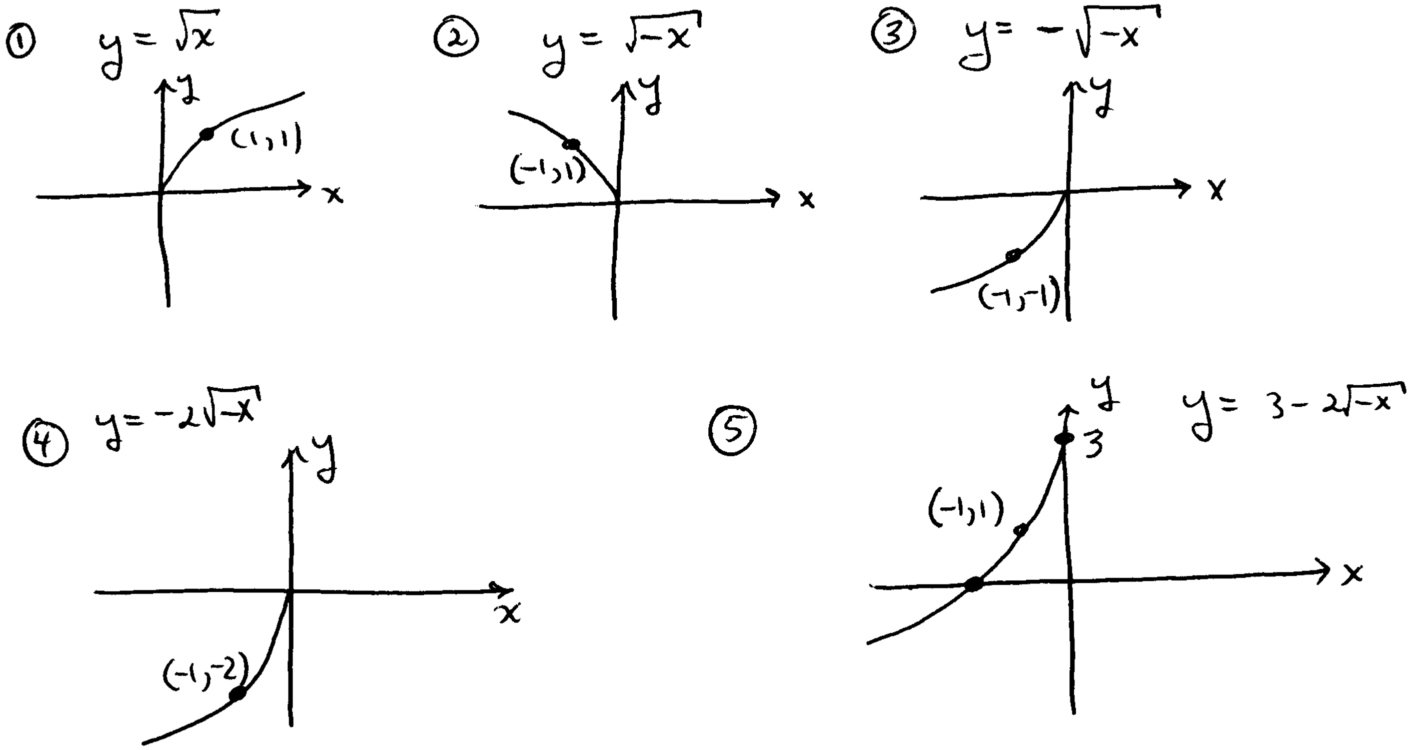
$= -(x-3)^2+16$

$\therefore$ vertex: $(3,16)$

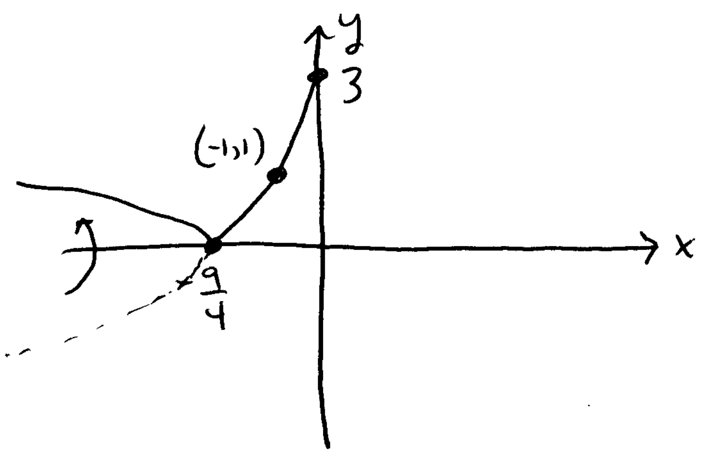
axis of symmetry: $x=3$.

Question 8:

(a)[5 points] Apply transformations to sketch the graph of $y = 3 - 2\sqrt{-x}$.



(b)[5 points] Using your result from (a) sketch the graph of $y = |3 - 2\sqrt{-x}|$. Label the x and y intercepts on your graph.



Question 9:

(a)[5 points] Let $f(x) = \dfrac{3x}{x-6}$. Find $f^{-1}(x)$ and state its domain and range.

$$y = \frac{3x}{x-6}$$

$$x = \frac{3y}{y-6}$$

$$xy - 6x = 3y$$

$$3y - xy = -6x$$

$$y(3-x) = -6x$$

$$y = \frac{-6x}{3-x}$$

$$y = \frac{6x}{x-3}$$

$$\therefore f^{-1}(x) = \frac{6x}{x-3}$$

domain of $f^{-1}(x)$: all real $x \neq 3$: $(-\infty, 3) \cup (3, \infty)$

range of $f^{-1}(x)$ = domain of $f(x)$
$= (-\infty, 6) \cup (6, \infty)$

(b)[3 points] Suppose $(-3, 1/2)$ is on the graph of $y = g(x)$ for some one-to-one function g. Evaluate

$$6g(-3) + 2g^{-1}(1/2)$$

$$= 6\left(\tfrac{1}{2}\right) + 2(-3)$$

$$= 3 - 6$$

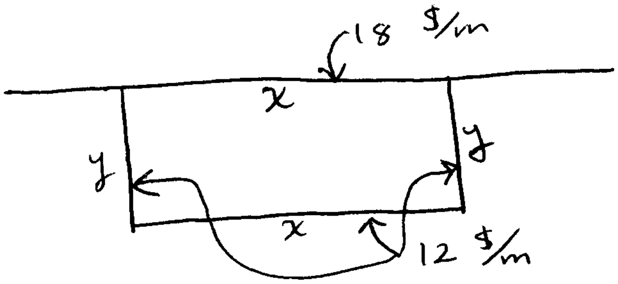
$$= -3$$

(c)[2 points] Let $F(x) = \dfrac{1 + (\sqrt{x}+2)^2}{\sqrt{x}+2}$. Find functions f and g so that $F = f \circ g$.

Let $g(x) = \sqrt{x} + 2$, $f(x) = \dfrac{1+x^2}{x}$

Question 10: A farmer wishes to construct a rectangular enclosure parallel to a straight road. the fencing for three sides of enclosure costs \$12 per metre, while the fencing for the side next to the road costs \$18 per metre since it must be taller than the other three sides. \$4800 is available for the project.

(a)[5 points] Let x represent the length of the side parallel to the road and $A(x)$ the area of the enclosure as a function of x. Find a formula for $A(x)$ and state the domain.



$\therefore\ 12x + 18x + 2(12)y = 4800$

$30x + 24y = 4800$

$y = \dfrac{4800 - 30x}{24}$

$= \dfrac{800 - 5x}{4}$

Area $= xy$

$\therefore\ A(x) = x\left(\dfrac{800-5x}{4}\right)$, $0 \le x \le 160$

(b)[5 points] Find the dimensions of the enclosure of maximum possible area.

Find x which maximizes $A(x)$ on $0 \le x \le 160$.

$$A(x) = x\left(\frac{800-5x}{4}\right) = \frac{-5}{4}x^2 + 200x$$

$$= \frac{-5}{4}\left[x^2 - 160x\right]$$

$$= \frac{-5}{4}\left[(x-80)^2 - 6400\right]$$

$$= \frac{-5}{4}(x-80)^2 + 8000.$$

$\therefore$ max. of $A(x)$ occurs at $x = 80$, $y = \dfrac{800-5(80)}{4} = 100$.

$\therefore$ enclosure should be 80m by 100 m.