Question 1:

(a)[4 points] Express as a single simplified fraction:

$$\frac{x}{x+1} + \frac{1}{x-1}$$

(b)[3 points] Expand and simplify:

$$2\left(\frac{a+b}{2}\right)^2 - \left(\frac{a^2+b^2}{2}\right)$$

(c)[3 points] Simplify using only positive exponents:

$$\left(\frac{(2x)^3 y^{-3}}{x^{-3}(2y)^{-2}}\right)$$

Question 2:

(a)[5 points] Solve and state your answer using interval notation

$$-3 \leq \frac{2}{3} - 5x \leq 2$$

(b)[5 points] Solve and state your answer using interval notation

$$x^2 + x > 12$$

Question 3:

(a)[4 points] Solve for x

$$\left|\frac{5x-3}{7}\right| = \frac{1}{2}$$

(b)[6 points] Solve and state your answer using interval notation

$$\left|\frac{8-11x}{-3}\right| \geq 5$$

Question 4:

(a)[5 points] The distance from $(-2, 5)$ to (a, a) is 9 units. Find all possible values of a.

(b)[5 points] Find the points of intersection of the graphs of $y = 2x^2 - 3x + 1$ and $3x - y + 9 = 0$.

Question 5:

(a)[4 points] Put the equation of the following circle in standard form and state the centre and radius

$$x^2 + y^2 - 3x + 5y = 11$$

(b)[3 points] Find the x and y intercepts of the graph of $y = \dfrac{(x^2-1)(x^2+2)}{x^2-3}$.

(c)[3 points] Find the zeros of $f(x) = x\sqrt{2x-7} - 7\sqrt{2x-7}$.

Question 6:

(a)[3 points] Find the domain of $f(x) = \dfrac{2\sqrt{9-x}}{x}$.

(b)[7 points] Factor completely

$$f(x) = x^4 + 2x^3 - 2x^2 - 6x - 3$$

Question 7:

(a) **[5 points]** Find the equation of the line through the midpoint of $(-1, -7)$ and $(7, 6)$ which is parallel to the line $5x - 3y - 4 = 0$.

(b) **[5 points]** Put the parabola $y = -x^2 + 6x + 7$ in standard form and state the vertex and axis of symmetry.

Question 8:

(a)[5 points] Apply transformations to sketch the graph of $y = 3 - 2\sqrt{-x}$.

(b)[5 points] Using your result from (a) sketch the graph of $y = |3 - 2\sqrt{-x}|$. Label the x and y intercepts on your graph.

Question 9:

(a)[5 points] Let $f(x) = \dfrac{3x}{x-6}$. Find $f^{-1}(x)$ and state its domain and range.

(b)[3 points] Suppose $(-3, 1/2)$ is on the graph of $y = g(x)$ for some one-to-one function g. Evaluate

$$6g(-3) + 2g^{-1}(1/2)$$

(c)[2 points] Let $F(x) = \dfrac{1 + (\sqrt{x}+2)^2}{\sqrt{x}+2}$. Find functions f and g so that $F = f \circ g$.

Question 10: A farmer wishes to construct a rectangular enclosure parallel to a straight road. the fencing for three sides of enclosure costs \$12 per metre, while the fencing for the side next to the road costs \$18 per metre since it must be taller than the other three sides. \$4800 is available for the project.

(a)[5 points] Let x represent the length of the side parallel to the road and $A(x)$ the area of the enclosure as a function of x. Find a formula for $A(x)$ and state the domain.

(b)[5 points] Find the dimensions of the enclosure of maximum possible area.

You may find some of the following formulas useful:

$$a^2 + b^2 = c^2$$

$$y = mx + b$$

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

$$acx^2 + (bc + ad)x + bd = (ax + b)(cx + d)$$

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$