

Question 1:

(a)[5 points] Find the equation of the line through $(1, -3)$ parallel to $2x - 5y + 4 = 0$.

$$2x - 5y + 4 = 0$$

$$5y = 2x + 4$$

$$y = \frac{2}{5}x + \frac{4}{5} \quad \left. \vphantom{y = \frac{2}{5}x + \frac{4}{5}} \right\} \text{slope } \frac{2}{5}$$

$$\therefore y - (-3) = \frac{2}{5}(x - 1)$$

$$\boxed{y + 3 = \frac{2}{5}(x - 1)}$$

or

$$\boxed{y = \frac{2}{5}x - \frac{17}{5}}$$

(b)[5 points] Find the points of intersection of the graphs of the linear functions

$$f(x) = 2x - 10 \quad \text{and} \quad g(x) = -3x - \frac{1}{2}$$

$$2x - 10 = -3x - \frac{1}{2}$$

$$5x = -\frac{1}{2} + \frac{20}{2}$$

$$5x = \frac{19}{2}$$

$$x = \frac{19}{10}$$

$$\therefore y = f\left(\frac{19}{10}\right) = 2\left(\frac{19}{10}\right) - 10 = \frac{19}{5} - \frac{50}{5} = -\frac{31}{5}$$

$$\therefore \text{point of intersection is } \left(\frac{19}{10}, -\frac{31}{5}\right)$$

Question 2:

- (a)[5 points] Express the quadratic function $f(x) = -x^2 + 6x - 10$ in standard form and sketch the graph of the function. Label the y -intercept and the vertex on your graph.

$$y = -x^2 + 6x - 10$$

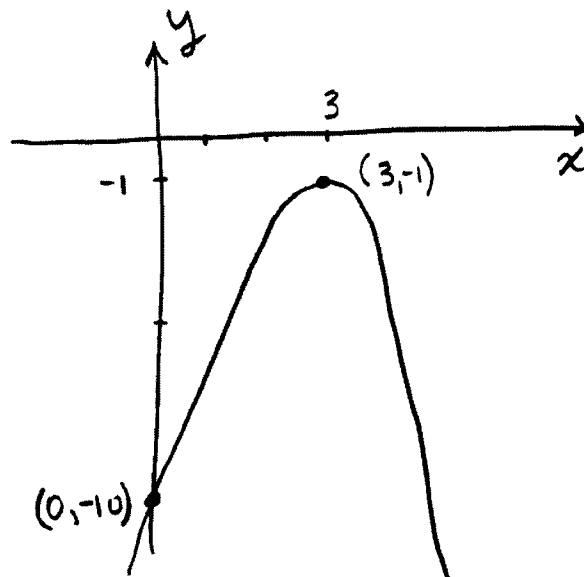
$$= -[x^2 - 6x + 10]$$

$$= -[(x-3)^2 - 9 + 10]$$

$$= -(x-3)^2 - 1$$

y -intercept : $(0, -10)$

Vertex : $(3, -1)$



- (b)[5 points] Find the the points of intersection of the graphs of

$$y = 2x - 2 \text{ and } y = 1 - x^2$$

$$2x - 2 = 1 - x^2$$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

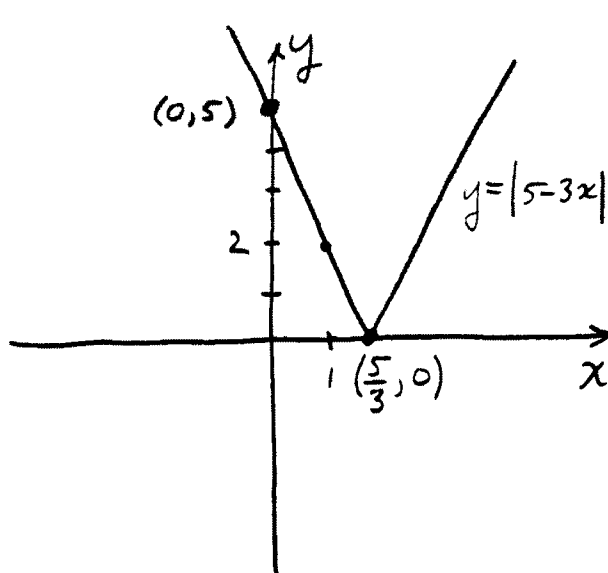
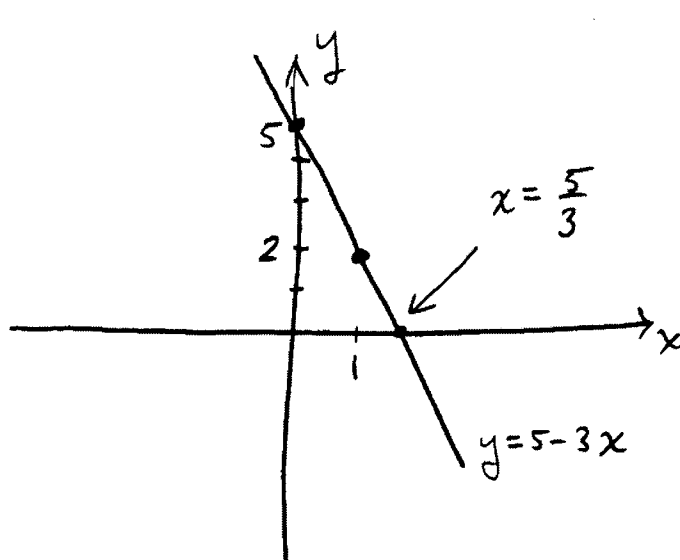
$$x = -3 \quad \left\{ \quad x = 1 \right.$$

$$\therefore \begin{array}{l} y = 2(-3) - 2 \\ \quad = -8 \end{array} \quad \left\{ \quad \begin{array}{l} y = 2(1) - 2 \\ \quad = 0 \end{array} \right.$$

$$\therefore (-3, -8), (1, 0)$$

Question 3:

(a)[5 points] Sketch the graph of $y = |5 - 3x|$. Label the x and y intercepts on your graph.



(b)[5 points] Let $f(x) = 2x + \frac{1}{x-1}$ and $g(x) = \frac{1}{x}$. Compute and simplify $(f \circ g)(x)$ and state the domain.

$$(f \circ g)(x) = f(g(x))$$

$$= 2\left(\frac{1}{x}\right) + \frac{1}{\left(\frac{1}{x}\right) - 1}$$

$$= \frac{2}{x} + \frac{x}{1-x}$$

$$= \frac{2(1-x) + x^2}{x(1-x)}$$

$$= \frac{x^2 - 2x + 2}{x(1-x)}$$

Domain must exclude $x=0$ and $x=1$, so domain is $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$.

Question 4:

- (a)[5 points] The one-to-one function $f(x) = \frac{x+1}{x+2}$ has domain $(-\infty, -2) \cup (-2, \infty)$ and range $(-\infty, 1) \cup (1, \infty)$. Find $f^{-1}(x)$ and state its domain and range.

$$y = \frac{x+1}{x+2}$$

$$x \leftrightarrow y:$$

$$x = \frac{y+1}{y+2}$$

$$xy + 2x = y + 1$$

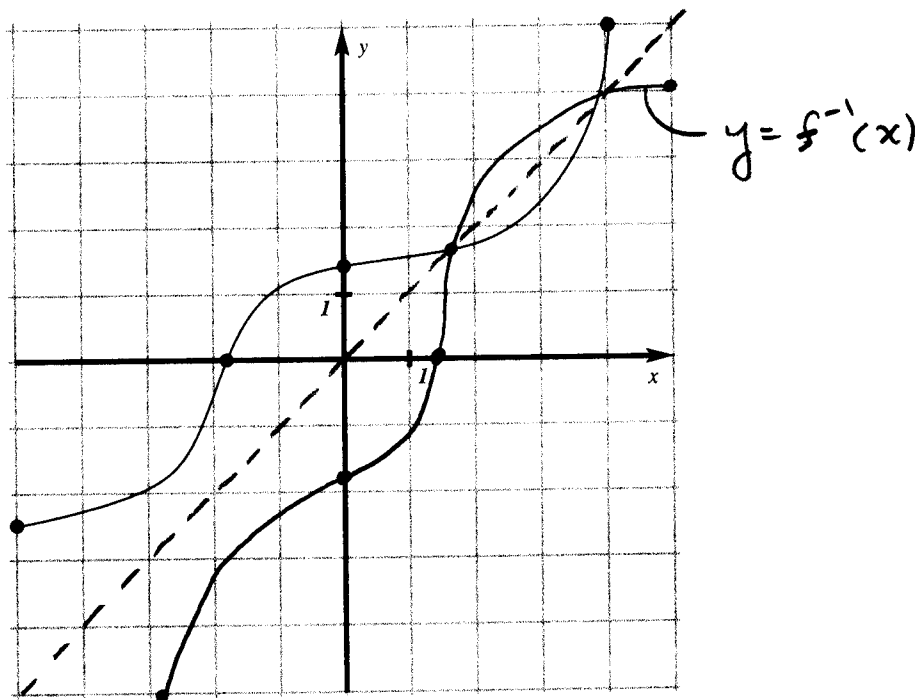
$$xy - y = 1 - 2x$$

$$y(x-1) = 1 - 2x$$

$$y = \frac{1-2x}{x-1}$$

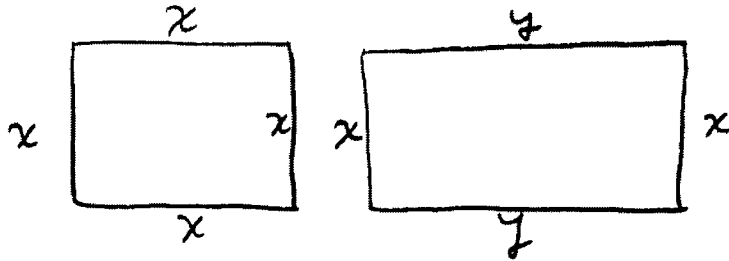
$$\therefore f^{-1}(x) = \frac{1-2x}{x-1}, \quad \text{domain : } (-\infty, 1) \cup (1, \infty) \\ \text{range : } (-\infty, -2) \cup (-2, \infty).$$

- (b)[5 points] Below is the graph $y = f(x)$ for some function f . Sketch the graph of $y = f^{-1}(x)$ on the same coordinate axes. Your graph must be accurate to receive full marks.



Question 5: A farmer wishes to build two enclosures using fencing. One enclosure must be a square, and the second must be a rectangle with the length of one side equal to the side length of the square. The two enclosures are not connected. 1000 m of fencing is available for the project.

(a)[5 points] Let x represent the side length of the square and $A(x)$ the total area of both enclosures as a function of x . Find a formula for $A(x)$.



$$6x + 2y = 1000 \text{ m}$$

$$A = x^2 + xy$$

$$6x + 2y = 1000$$

$$y = \frac{1000 - 6x}{2} = 500 - 3x$$

$$\therefore A(x) = x^2 + x(500 - 3x), \quad 0 \leq x \leq \frac{500}{3}$$

(b)[5 points] Find the dimensions of the square and rectangle which enclose the largest area possible. (The solution may appear obvious to you, but prove it using $A(x)$ from part (a).)

$$\text{Maximize } A(x) = x^2 + x(500 - 3x) \text{ on } 0 \leq x \leq \frac{500}{3}$$

$$A(x) = x^2 + 500x - 3x^2$$

$$= -2x^2 + 500x$$

$$= -2[x^2 - 250x]$$

$$= -2[(x - 125)^2 - 125^2]$$

$$= -2(x - 125)^2 + 31,250 \quad \text{which has a maximum at } x = 125.$$

$$\therefore x = 125 \text{ m and } y = 500 - 3(125) = 125 \text{ m.}$$