Question 1:

(a)[**5 points**] Solve for w:

$$|2-11w| = 3$$

$2 - 11w = 3$ or $2 - 11w = -3$

$-11w = 1$ $-11w = -5$

$w = \frac{-1}{11}$ $w = \frac{5}{11}$

$\therefore w = \frac{-1}{11}$ or $\frac{5}{11}$

(b)[**5 points**] Solve for x and state your answer using interval notation:

$$\left|\frac{2-5x}{3}\right| \geq 5$$

$\frac{2-5x}{3} \geq 5$ or $\frac{2-5x}{3} \leq -5$

$2-5x \geq 15$ $2-5x \leq -15$

$-5x \geq 13$ $-5x \leq -17$

$x \leq -\frac{13}{5}$ $x \geq \frac{17}{5}$

$\therefore \left(-\infty, \frac{-13}{5}\right] \cup \left[\frac{17}{5}, \infty\right)$

Question 2:

(a)[5 points] Find both (i) the distance and (ii) midpoint between the points $A(-1, 4)$ and $B(3, -1)$.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{(-1) + 3}{2}, \frac{4 + (-1)}{2}\right)$$

$$= \left(1, \frac{3}{2}\right)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3 - (-1))^2 + (-1 - 4)^2}$$

$$= \sqrt{16 + 25}$$

$$= \sqrt{41}$$

(b)[5 points] The point (a, a) is a distance 5 units from the point $(3, -4)$. Find all possible values of a.

$$5 = \sqrt{(3-a)^2 + (-4-a)^2}$$

$$25 = (3-a)^2 + (-4-a)^2$$

$$25 = 9 - 6a + a^2 + 16 + 8a + a^2$$

$$2a^2 + 2a = 0$$

$$2a(a+1) = 0$$

$$2a = 0, \quad a + 1 = 0$$

$$a = 0, \quad a = -1$$

Question 3:

(a)[5 points] Complete the square and state the centre and radius of the following circle:

$$x^2 + y^2 - 8x + 10y + 37 = 0$$

$$x^2 - 8x \quad + y^2 + 10y \quad + 37 = 0$$

$$(x-4)^2 - 16 + (y+5)^2 - 25 + 37 = 0$$

$$(x-4)^2 + (y+5)^2 = 4$$

$\therefore$ centre $(4,-5)$, radius $= 2$

(b)[5 points] Give the equation of the circle with centre $\left(\frac{1}{2}, \frac{-1}{2}\right)$ that passes through the point $\left(2, \frac{3}{2}\right)$.

$$r = \sqrt{\left(\tfrac{1}{2} - 2\right)^2 + \left(\tfrac{-1}{2} - \tfrac{3}{2}\right)^2}$$

$$= \sqrt{\frac{9}{4} + \frac{16}{4}}$$

$$= \sqrt{\frac{25}{4}}$$

$$= \frac{5}{2}$$

$\therefore$ equation is $\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{25}{4}$

Question 4: This question concerns the equation $x = 2y^2 - 18$.

(a)[**4 points**] Find the x and y intercepts of the graph of the equation.

x-intercept:

$x = 2(0)^2 - 18$

$x = -18$

$\therefore (-18, 0)$

y-intercept:

$0 = 2y^2 - 18$

$2y^2 = 18$

$y^2 = 9$

$y = 3, -3$

$\therefore (0,3), (0,-3)$

(b)[**6 points**] Determine if the graph of the equation possesses symmetry with respect to the x-axis, the y-axis, or the origin.

x-axis:

$x = 2y^2 - 18$

$y \leftrightarrow -y$:

$x = 2(-y)^2 - 18$

$\therefore x = 2y^2 - 18$

equivalent, so graph possesses x-axis symmetry

y-axis:

$x = 2y^2 - 18$

$x \leftrightarrow -x$:

$-x = 2y^2 - 18$

not equivalent so graph does not possess y-axis symmetry.

Origin:

$x = 2y^2 - 18$

$(x,y) \leftrightarrow (-x,-y)$

$-x = 2(-y)^2 - 18$

$-x = 2y^2 - 18$

not equivalent, so graph does not possess symmetry about the origin

Question 5:

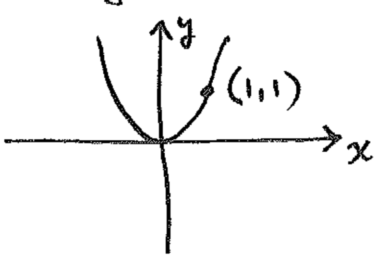
(a)[5 points] Find the zeros of $f(x) = 2(x-3)^2 - 4$.

$$2(x-3)^2 - 4 = 0$$
$$2(x-3)^2 = 4$$
$$(x-3)^2 = 2$$
$$x - 3 = \sqrt{2}, -\sqrt{2}$$
$$x = 3+\sqrt{2}, 3-\sqrt{2}$$

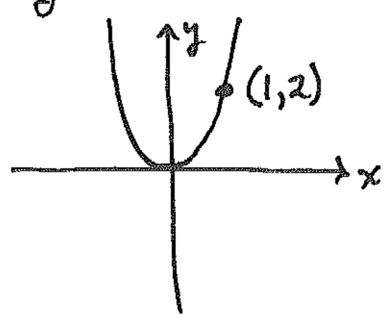
(b)[5 points] Sketch the graph of $f(x) = 2(x-3)^2 - 4$ by applying transformations to one of the basic functions we saw in class. Show at least one point on your final graph, and also indicate the scale on the x and y axes.

Let $y = f(x)$.

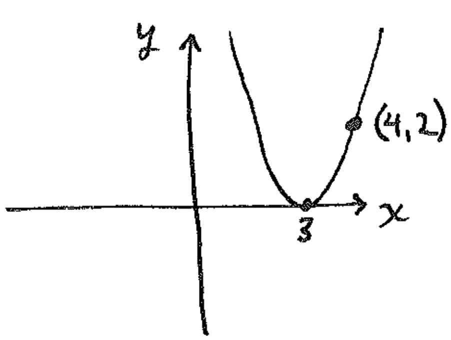
① $y = x^2$:



② $y = 2x^2$:



③ $y = 2(x-3)^2$



④ $y = 2(x-3)^2 - 4$