(1)[5 points]   Sketch the graph of $y = |x^2 - 2x|$ and give the x and y intercepts.

$\underline{x\text{-intercepts}}$:

$0 = |x^2 - 2x|$

$0 = x^2 - 2x$

$0 = x(x-2)$

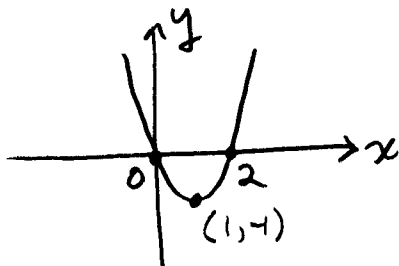$x = 0, 2$,

$\therefore (0,0) , (2,0)$

$\underline{y\text{-intercepts}}$:
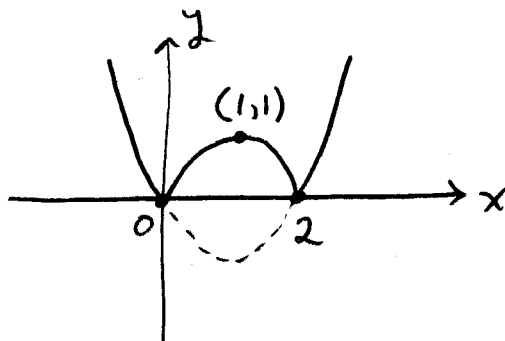
$y = |0^2 - 2(0)|$

$y = 0$

$\therefore (0,0)$

For sketch, $y = x^2 - 2x = (x-1)^2 - 1$, which has graph

$\therefore y = |x^2 - 2x|$ has graph

(2)[5 points]   Let $f(x) = \dfrac{1}{2x-1}$ and $g(x) = x^2 + 1$. Find $f \circ g$ as well as $g \circ f$ and state the domains for each.

$$(f \circ g)(x) = f(g(x)) = \frac{1}{2(x^2+1)-1} = \frac{1}{2x^2+1}$$

For domain, we require that $2(x^2+1)-1 \neq 0$.

Solving $2(x^2+1)-1 = 0$

$$2x^2+1 = 0$$
$$x^2 = \frac{-1}{2}$$, which has no real solutions.

∴ domain of $f \circ g$ is $(-\infty, \infty)$

$$(g \circ f)(x) = g(f(x)) = (f(x))^2 + 1 = \left(\frac{1}{2x-1}\right)^2 + 1$$

For domain, we require that $2x-1 \neq 0$.

Solving $2x-1 = 0$, we find $x = \frac{1}{2}$.

∴ domain of $g \circ f$ is $(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$.

(3)[5 points]   Let $F(x) = (x^2 + 2x)^2 + 5\sqrt{x^2 + 2x}$. Find functions f and g such that $F = f \circ g$.

Let $g(x) = x^2 + 2x$

$$f(x) = x^2 + 5\sqrt{x}$$

Check: $(f \circ g)(x) = f(g(x))$

$$= (g(x))^2 + 5\sqrt{g(x)}$$

$$= (x^2+2x)^2 + 5\sqrt{x^2+2x}$$