Question 1:

(a)[5 points] Find the equation of the line through $(1, -3)$ parallel to $2x - 5y + 4 = 0$.

$$2x - 5y + 4 = 0$$

$$5y = 2x + 4$$

$$y = \frac{2}{5}x + \frac{4}{5} \Big\} \text{ slope } \frac{2}{5}$$

$$\therefore \quad y - (-3) = \frac{2}{5}(x-1)$$

$$\boxed{y + 3 = \frac{2}{5}(x-1)}$$

or $\boxed{y = \frac{2}{5}x - \frac{17}{5}}$

(b)[5 points] Find the points of intersection of the graphs of the linear functions

$$f(x) = 2x - 10 \quad \text{and} \quad g(x) = -3x - \frac{1}{2}$$

$$2x - 10 = -3x - \frac{1}{2}$$

$$5x = \frac{-1}{2} + \frac{20}{2}$$

$$5x = \frac{19}{2}$$

$$x = \frac{19}{10}$$

$$\therefore \quad y = f\left(\frac{19}{10}\right) = 2\left(\frac{19}{10}\right) - 10 = \frac{19}{5} - \frac{50}{5} = \frac{-31}{5}$$

$\therefore$ point of intersection is $\left(\frac{19}{10}, \frac{-31}{5}\right)$

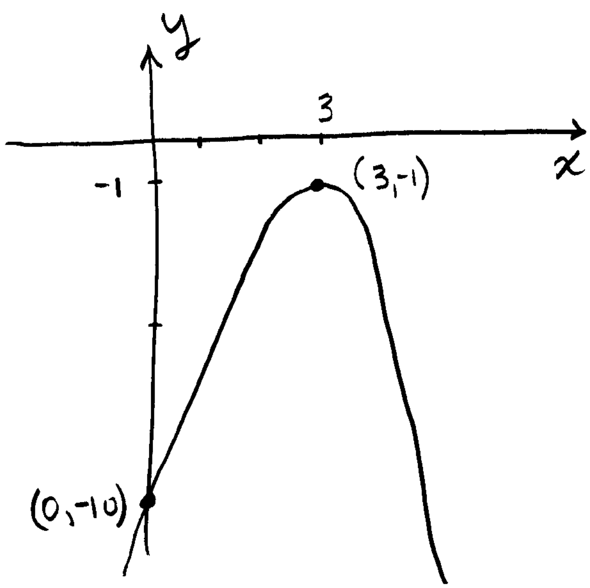
Question 2:

(a)[5 points] Express the quadratic function $f(x) = -x^2 + 6x - 10$ in standard form and sketch the graph of the function. Label the y-intercept and the vertex on your graph.

$$y = -x^2 + 6x - 10$$
$$= -[x^2 - 6x + 10]$$
$$= -[(x-3)^2 - 9 + 10]$$
$$= -(x-3)^2 - 1$$

y- intercept : $(0,-10)$

Vertex : $(3,-1)$



(b)[5 points] Find the the points of intersection of the graphs of

$$y = 2x - 2 \quad \text{and} \quad y = 1 - x^2$$

$$2x - 2 = 1 - x^2$$
$$x^2 + 2x - 3 = 0$$
$$(x+3)(x-1) = 0$$

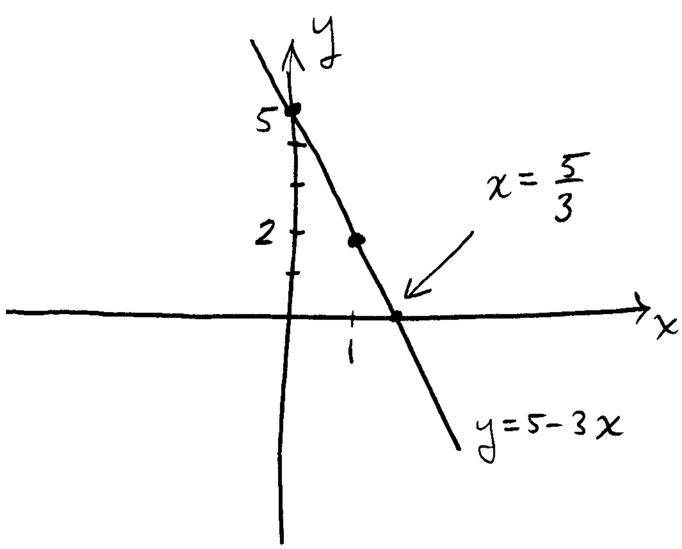
$x = -3$; $x = 1$

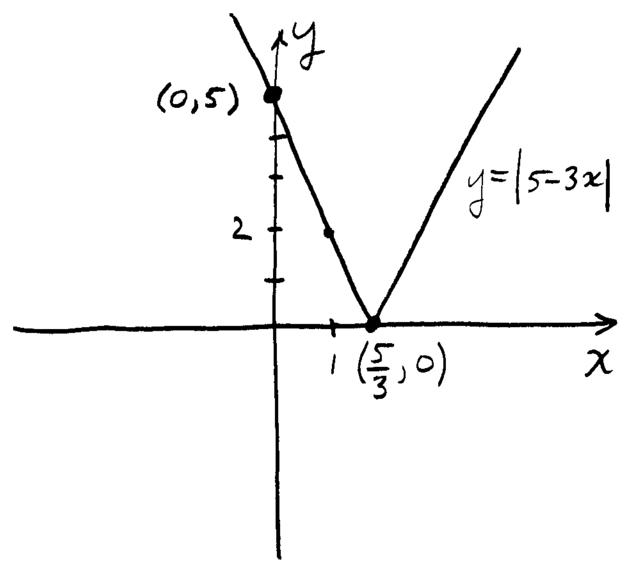
$\therefore$ $y = 2(-3) - 2 = -8$ | $y = 2(1) - 2 = 0$

$\therefore (-3,-8)$, $(1,0)$

Question 3:

(a)[5 points] Sketch the graph of $y = |5 - 3x|$. Label the x and y intercepts on your graph.





(b)[5 points] Let $f(x) = 2x + \frac{1}{x-1}$ and $g(x) = \frac{1}{x}$. Compute and simplify $(f \circ g)(x)$ and state the domain.

$$(f \circ g)(x) = f(g(x))$$

$$= 2\left(\frac{1}{x}\right) + \frac{1}{\left(\frac{1}{x}\right) - 1}$$

$$= \frac{2}{x} + \frac{x}{1-x}$$

$$= \frac{2(1-x) + x^2}{x(1-x)}$$

$$= \frac{x^2 - 2x + 2}{x(1-x)}$$

Domain must exclude $x = 0$ and $x = 1$, so domain is $(-\infty, 0) \cup (0,1) \cup (1, \infty)$.

Question 4:

(a)[5 points] The one-to-one function $f(x) = \dfrac{x+1}{x+2}$ has domain $(-\infty,-2)\cup(-2,\infty)$ and range $(-\infty,1)\cup(1,\infty)$. Find $f^{-1}(x)$ and state its domain and range.

$$y = \frac{x+1}{x+2}$$

$x \leftrightarrow y$:

$$x = \frac{y+1}{y+2}$$

$$xy + 2x = y + 1$$

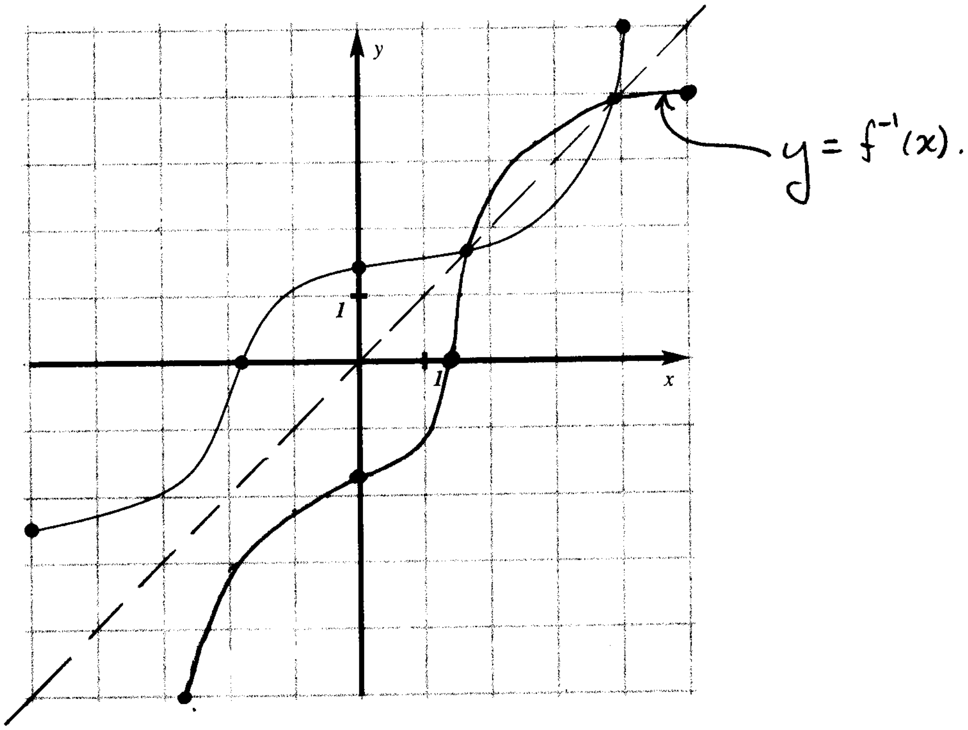
$$xy - y = 1 - 2x$$

$$y(x-1) = 1 - 2x$$

$$y = \frac{1-2x}{x-1}$$

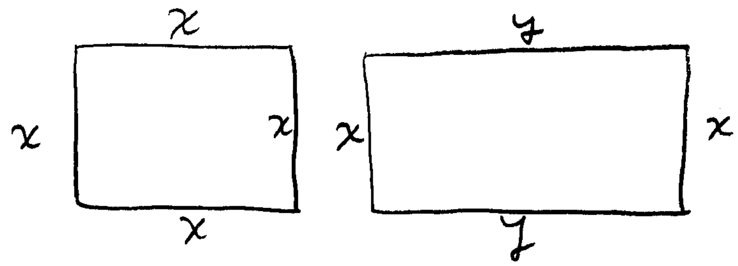
$\therefore f^{-1}(x) = \dfrac{1-2x}{x-1}$, domain : $(-\infty,1)\cup(1,\infty)$
range : $(-\infty,-2)\cup(-2,\infty)$.

(b)[5 points] Below is the graph $y = f(x)$ for some function f. Sketch the graph of $y = f^{-1}(x)$ on the same coordinate axes. Your graph must be accurate to receive full marks.



Question 5: A farmer wishes to build two enclosures using fencing. One enclosure must be a square, and the second must be a rectangle with the length of one side equal to the side length of the square. The two enclosure are not connected. 1000 m of fencing is available for the project.

(a)[5 points] Let x represent the side length of the square and $A(x)$ the total area of both enclosures as a function of x. Find a formula for $A(x)$.



$6x + 2y = 1000\text{m}$

$A = x^2 + xy$

$$6x + 2y = 1000$$

$$y = \frac{1000 - 6x}{2} = 500 - 3x$$

$$\therefore A(x) = x^2 + x(500 - 3x), \quad 0 \le x \le \frac{500}{3}.$$

(b)[5 points] Find the dimensions of the square and rectangle which enclose the largest area possible. (The solution may appear obvious to you, but prove it using $A(x)$ from part (a).)

Maximize $A(x) = x^2 + x(500 - 3x)$ on $0 \le x \le \frac{500}{3}$

$$\begin{aligned} A(x) &= x^2 + 500x - 3x^2 \\ &= -2x^2 + 500x \\ &= -2\left[x^2 - 250x\right] \\ &= -2\left[(x-125)^2 - 125^2\right] \\ &= -2(x-125)^2 + 31{,}250 \end{aligned}$$

which has a maximum at $x = 125$.

$\therefore$ $x = 125$ m and $y = 500 - 3(125) = 125$ m.