Question 1:

(a)[5 points] Solve for w:

$$|2 - 11w| = 3$$

$$2 - 11w = 3 \qquad \text{or} \qquad 2 - 11w = -3$$

$$-11w = 1 \qquad\qquad\qquad -11w = -5$$

$$w = \frac{-1}{11} \qquad\qquad\qquad w = \frac{5}{11}$$

$$\therefore \; w = \frac{-1}{11} \; \text{or} \; \frac{5}{11}$$

(b)[5 points] Solve for x and state your answer using interval notation:

$$\left| \frac{2 - 5x}{3} \right| \geq 5$$

$$\frac{2-5x}{3} \geq 5 \qquad\qquad \text{or} \qquad\qquad \frac{2-5x}{3} \leq -5$$

$$2 - 5x \geq 15 \qquad\qquad\qquad\qquad 2 - 5x \leq -15$$

$$-5x \geq 13 \qquad\qquad\qquad\qquad -5x \leq -17$$

$$x \leq -\frac{13}{5} \qquad\qquad\qquad\qquad x \geq \frac{17}{5}$$

$$\therefore \; \left(-\infty, \frac{-13}{5} \right] \cup \left[\frac{17}{5}, \infty \right)$$

Question 2:

(a)[5 points] Find both (i) the distance and (ii) midpoint between the points $A(-1,4)$ and $B(3,-1)$.

$$M = \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{(-1) + 3}{2} , \frac{4 + (-1)}{2} \right)$$

$$= \left(1, \frac{3}{2} \right)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3 - (-1))^2 + (-1 - 4)^2}$$

$$= \sqrt{16 + 25}$$

$$= \sqrt{41}$$

(b)[5 points] The point (a, a) is a distance 5 units from the point $(3, -4)$. Find all possible values of a.

$$5 = \sqrt{(3-a)^2 + (-4-a)^2}$$

$$25 = (3-a)^2 + (-4-a)^2$$

$$25 = 9 - 6a + a^2 + 16 + 8a + a^2$$

$$2a^2 + 2a = 0$$

$$2a(a+1) = 0$$

$$2a = 0 \quad , \quad a + 1 = 0$$

$$a = 0 \quad , \quad a = -1$$

Question 3:

(a)[5 points] Complete the square and state the centre and radius of the following circle:

$$x^2 + y^2 - 8x + 10y + 37 = 0$$

$$x^2 - 8x + y^2 + 10y + 37 = 0$$

$$(x-4)^2 - 16 + (y+5)^2 - 25 + 37 = 0$$

$$(x-4)^2 + (y+5)^2 = 4$$

$$\therefore \text{ centre } (4, -5), \quad \text{radius} = 2$$

(b)[5 points] Give the equation of the circle with centre $\left(\frac{1}{2}, \frac{-1}{2}\right)$ that passes through the point $\left(2, \frac{3}{2}\right)$.

$$r = \sqrt{\left(\frac{1}{2} - 2\right)^2 + \left(\frac{-1}{2} - \frac{3}{2}\right)^2}$$

$$= \sqrt{\frac{9}{4} + \frac{16}{4}}$$

$$= \sqrt{\frac{25}{4}}$$

$$= \frac{5}{2}$$

$$\therefore \text{ equation is } \left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{25}{4}$$

Question 4: This question concerns the equation $x = 2y^2 - 18$.

(a)[4 points] Find the x and y intercepts of the graph of the equation.

x-intercept:

$x = 2(0)^2 - 18$

$x = -18$

$\therefore (-18, 0)$

y-intercept:

$0 = 2y^2 - 18$

$2y^2 = 18$

$y^2 = 9$

$y = 3, -3$

$\therefore (0, 3), (0, -3)$

(b)[6 points] Determine if the graph of the equation possesses symmetry with respect to the x-axis, the y-axis, or the origin.

x-axis:

$x = 2y^2 - 18$

$y \leftrightarrow -y:$

$x = 2(-y)^2 - 18$

$\therefore x = 2y^2 - 18$

equivalent, so graph possesses x-axis symmetry

y-axis:

$x = 2y^2 - 18$

$x \leftrightarrow -x:$

$-x = 2y^2 - 18$

not equivalent so graph does not possess y-axis symmetry.

Origin:

$x = 2y^2 - 18$

$(x, y) \leftrightarrow (-x, -y)$

$-x = 2(-y)^2 - 18$

$-x = 2y^2 - 18$

not equivalent, so graph does not possess symmetry about the origin

Question 5:

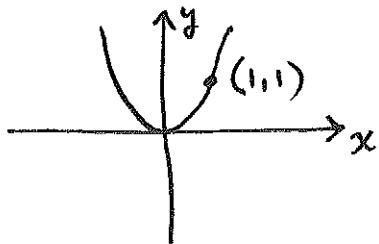(a)[5 points] Find the zeros of $f(x) = 2(x-3)^2 - 4$.

$$2(x-3)^2 - 4 = 0$$
$$2(x-3)^2 = 4$$
$$(x-3)^2 = 2$$
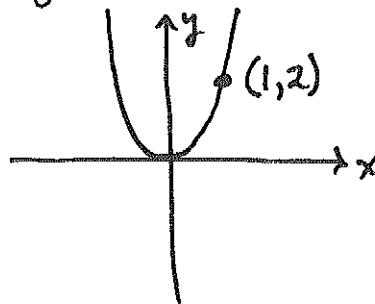$$x-3 = \sqrt{2}, -\sqrt{2}$$
$$x = 3 + \sqrt{2}, \ 3 - \sqrt{2}$$

(b)[5 points] Sketch the graph of $f(x) = 2(x-3)^2 - 4$ by applying transformations to one of the basic functions we saw in class. Show at least one point on your final graph, and also indicate the scale on the x and y axes.
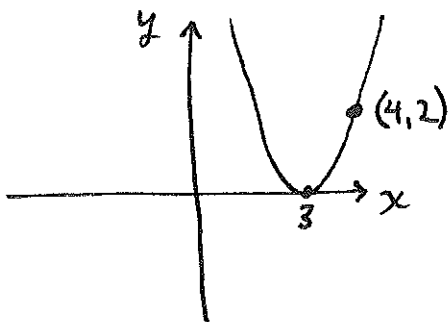
Let $y = f(x)$.

① $y = x^2$:

② $y = 2x^2$:

③ $y = 2(x-3)^2$

④ $y = 2(x-3)^2 - 4$