Math 191 Sec F0501/F0502
Practice Problems for Final Exam

Short Problems

1. Find the effective rate that corresponds to an interest rate of 5% compounded daily.
2. At what nominal rate of interest compounded semiannually will an investment double in 20 years?
3. Find the present value of \$3000 due after five years if the interest rate is 9.6% compounded semiannually.
4. How much must be invested at an interest rate of 7.5% compounded monthly to have \$5000 in five years?
5. Find the present value of an annuity of \$200 per month for 7.5 years at an interest rate of 7% compounded montly.
6. Find all values of x for which the curve $y = x^3 + x^2$ has slope 1.
7. Find all points x for which the curve $y = 2x^2 + 3x + 5$ has a tangent line that is perpendicular to the line $y = x + 15$.
8. Given average cost $\bar{c} = 0.001q^2 - 0.2q + 11 + \dfrac{15,000}{q}$, find the marginal cost function.
9. Find the relative rate of change of y with respect to x when $x = -1$ for $x^3 - 2x^2$.
10. Given the demand equation $p = 400 - q^2$ find the marginal revenue function.
11. Find the slope of the curve $y = \dfrac{2x+5}{x-3}$ at the point $x = 4$.
12. Given the demand equation $p = \dfrac{400}{q+10}$, find the marginal revenue when $q = 10$.
13. Given that m employees produce $q = 100m + m^2$ units of product per day, and the demand equation $p = \dfrac{3000}{q+100}$, find the marginal revenue product when $m = 10$.
14. Find y' where $y = \ln\left(\ln(2x+3)\right)$.
15. Given the revenue function $R(q) = \dfrac{3000q}{\ln(5q+20)}$ find the marginal revenue when $q = 5$.
16. Let $y = \dfrac{e^{2x+1}}{(1-2x)^2}$. Find y' when $x = 1$.
17. Given cost function $\dfrac{10,000e^{q/900}}{q}$, what is the marginal cost at a production level of 900?
18. Given demand equation $p = \dfrac{400}{q+2}$ find the point elasticity of demand when $q = 100$.
19. Find point elasticity of demand when $p = 24$ for the demand equation $(p+1)\sqrt{q+3} = 1000$.
20. Find y' if $\ln(xy) + y = 2$.

21. Find $\dfrac{dy}{dx}$ where $xy^2 = e^x + y$.
22. Using Newton's method to solve $x^4 - 6x + 3 = 0$, we start with $x_0 = 1$. Find x_1.
23. Using Newton's method to find the point of intersection of $y = \ln x$ and $y = 2 - x$, we start with $x_0 = 1$. Find x_1.
24. Find y'' in $e^y + y + x = 2$ and give your answer in terms of x and y only.
25. Find all critical values of $\dfrac{x^2+9}{x}$.
26. Find intervals of increase/decrease of $\dfrac{x^2}{e^x}$.
27. Find the absolute extrema of $\dfrac{x^2}{e^x}$ on the interval $[0, 3]$.
28. Find the absolute extrema of $f(x) = 4 + x^2 - x^3$ on the interval $[-1, 1]$.
29. Find the inflection points of $f(x) = e^x(x + 4)$.
30. Determine the intervals of concavity for $f(x) = \dfrac{x^2}{e^x}$
31. Write the linear approximation of $f(x) = (x + 1)e^{x/2}$ about $x = 0$.
32. Use a linear approximation to estimate $\ln(1.1)$.
33. Compute y' where $y = \ln(x^2)\cos(\pi x^4)$.
34. Find the absolute maximum of $f(x) = e^{\sin x}$ on the interval $[0, \pi]$.
35. What is the derivative of $\cos x$ if x is measured in degrees?

Longer Problems

1. (a) Give the definition of the derivative of the function $y = f(x)$.

   (b) Use the definition of the derivative to find $f'(x) = \dfrac{1}{2x} + 1$.

2. Find the derivative of $y = \ln\left(\dfrac{1-e^{x^2}}{1+e^{2x}}\right)$.

3. Find the equation of the tangent line to the curve

$$x^3 + x^2y^3 + \frac{y^4}{x} = 3$$

   at the point $(1, 1)$.

4. A storage shed is to be built in the shape of a box with a square base. It is to have a volume of 150 cubic metres. The concrete for the base costs \$4 per square metre, the material for the roof costs \$2 per square metre, and the material for the sides cost \$2.50 per square metre. Find the dimensions of the most economical shed.

5. Use y' and y'' to aid in graphing the function $y = x(x+4)^3$. In particular, you should identify all local (i.e. relative) maxima and minima, any inflection points and all asymptotes.

6. The number of people contracting a certain disease is growing exponentially. If 500 people have the disease on January 1, 1997 and the number of cases doubles every 18 months, how many people will have the disease on January 1, 2004?

7. Suppose a young couple takes out a \$100,000 mortgage to buy a condo with monthly payments for 25 years. The first payment occurs after one month and the last payment at the end of 25 years. What are the monthly payments if the nominal interest rate on the mortgage is 9% compounded monthly?

8. Consider the curve defined by $(x^2 + y^2)^3 = 8x^2y^2$.

   (a) Find the equation of the tangent line to this curve at the point $(1, -1)$. Write your answer in the form $y = mx + b$.

   (b) Approximate the value of y for $x = 0.8$.

9. A zero-coupon bond is a bond that is sold for less than its face value and has no periodic interest payments. Instead, the bond is redeemed for its face value at maturity. Thus, in a sense, interest is paid at maturity. Suppose that a zero-coupon bond sells for \$220 and can be redeemed in 14 years for its face value of \$1,000.

   (a) What is the nominal interest rate of the bond under semiannual compounding?

   (b) What is the nominal interest rate of the bond under continuous compounding?

   (c) What is the doubling time under the interest rate scheme in part (b)?

10. Suppose the demand equation for a given product is $q = \dfrac{60}{p} + \ln\left(65 - p^3\right)$ for $0 < p < 65^{1/3}$, where p is the unit price and q is the quantity demanded.

    (a) Find the elasticity of demand when $p = 4$.

    (b) Use the result of part (a) to estimated the percentage change in demand if the price decreases by 0.2% from \$4.

11. Use y' and y'' to aid in graphing the function $y = (x+1)e^{-x}$. In particular, you should identify all relative and absolute maxima and minima, any inflection points and all asymptotes. No credit will be given for the graph without the accompanying work.

12. Suppose that the cost of publishing a small book is \$10,000 to set up the annual press run, plus \$8 for each book printed. The publisher sold 7,000 copies last year at \$13 each, but sales dropped to 5,000 copies this year when the price was raised to \$15 per copy. If the demand function is linear, then how many copies should be printed, and what should be the selling price of each copy, in order to maximize the profit?

13. Find all points on the graph of $y = x^2 + 1$ where the tangent line passes through the point $(10, 37)$.

14. Compute y' by implicit differentiation: $y - x = \ln y + \ln x$.

15. Suppose you take out a 20 year mortgage on a commercial property for $500,000. The interest rate for the mortgage is 6% with monthly compounding. The equal payments on the mortgage are made monthly with the first payment being made 5 years and one month from the start of the mortgage, and the last payment at the end of the 20 years for a total of 180 payments. What is the amount of each payment?

16. Let the cost C in dollars of producing q items be given by $C(q) = 24q + 21,900$. Assume that $q = 300$ items will be demanded when the price of each item $p = 140$. Finally, suppose that with every increase in price of $1 the number of items demanded will decrease by 5. Use calculus to determine how the items should be priced to assure maximum profit.

17. Suppose the demand equation for a given product is $q = \sqrt{2500 - p^2}$ for $0 \leq p \leq 50$, where q is the number of items demanded and p is the price of a single item.

    (a) Find the elasticity of demand and indicate where it is inelastic and elastic.

    (b) Use the elasticity of demand in part (a) to estimate the percentage change in demand when the price increases by 0.5% from $40.

18. Find the equation of the tangent line to the curve $y = \dfrac{x}{1+x^2}$ at the point $(2, 2/5)$. Write your answer in the form $y = mx + b$.

19. The total cost in dollars of producing q grams of a certain chemical is given by $C(q) = 10q^2+40$, where $q > 0$. Use calculus to find the value of q which minimizes the average cost $\dfrac{C(q)}{q}$.

20. Find the slope of the tangent line to the curve $\dfrac{x}{y} + y^2 = \dfrac{5x^2}{4}$ at the point $(2, 2)$.

21. Over what interval or intervals is the function $x^2 \ln x$ increasing?

22. Over what interval or intervals is the function xe^{3x} concave down?

23. Assume that a company faces a demand equation for its product given by $p + 0.003q = 5$, where q is the number of kilograms demanded, and p is the price per kilogram (in dollars). Suppose further that the total cost C of producing q kilograms is given by $C(q) = 30 + 1.1q$ Determine the values of q and p which maximize the profit.

24. The total cost of making x items at factory X and y items at factory Y is given by $C(x, y) = 2x^2 + xy + y^2 + 500$. Assume that it is desired to make 200 items in total. Find the values of x and y which minimize the total cost.

25. Use a sketch to find the number of points of intersection of the curves $y = e^x$ and $y = 4 - x^2$. Now suppose you are using Newton's Method to compute the x-coordinate of one of the points of intersection. If you start with $x_0 = 1$, find x_2.

26. The function f is differentiable everywhere. It is known that $f(2.95) = 10.53$ and that $f(3) = 10.50$. Use this information to estimate $f'(3)$.

27. Find the equation of the tangent line to the curve $xe^{xy} + y = x^2$ at the point $(1, 0)$. Write your answer in the form $y = mx + b$.

28. Find the value of x where the slope of the tangent line to the function $y = x^3 \ln x$ is a minimum.

29. A broker offers you a choice of two ten-year investment opportunities. The first pays a nominal interest rate of 6% for the first five years with semiannual compounding, followed by a nominal interest rate of 7% for the last five years with continuous compounding. The second offers a nominal interest rate of 6.6% with yearly compounding for the full ten years. Which of the two investment opportunities is better?

30. The function $y = y(x)$ is defined implicitly by $e^{xy}+x+y = 1$. It is desired to find y when $x = 1$. Using Newton's Method with an initial approximation of $y_0 = 0$, find the next approximation y_1.