Review questions for Math 151 Final Exam **November 2005**

Q1. Simplify: (a) $\frac{3}{4}-\frac{7}{12}+\frac{3}{8}-\frac{5}{6}$ (b) $\frac{q}{r}+\frac{r}{p}-\frac{p}{qr}$ (c) $\frac{1}{x}\left(\frac{1}{z}-\frac{1}{y}\right)-\frac{1}{z}\left(\frac{1}{x}-\frac{1}{y}\right)$ (d) $\frac{(125\,a^3\,b^3)^{\left(\frac{2}{3}\right)}}{(25\,a\,b)^{\left(\frac{1}{2}\right)}}$

Q2. Simplify using only positive exponents.

(a) $(x^{(-1)}+y^{(-1)})^{(-1)}$ (b) $\frac{(x^3\,y^{(-2)})^6}{(y^{(-5)}\,x^{(-2)})^{(-3)}}$ (c) $\frac{v^{(-3)}-u^{(-3)}}{v^{(-2)}-u^{(-2)}}$.

Q3. (a) Evaluate the following expression and leave your answer in scientific notation and rounded to 3 significant digits.

$$\frac{2.58\,.\,10^{12}}{3.145\,.\,10^{(-7)}-3.52\,.\,10^{(-8)}}$$

(b) After a 55% reduction in price Hannah's new skates cost \$108. What was the original price?

Q4. Solve for x: (a) $\frac{1}{6}x+\frac{1}{3}=3x-\frac{1}{4}$ (b) $2x^2+9x-5=0$ (c) $x^2+4x-9=0$

(d) $\frac{x}{\sqrt{16-x^2}}=1$ (e) $\frac{3}{x}-\frac{3}{x+2}=2$ (f) $(\sqrt{x}+2)^3-64=0$.

Q5. (a) Find the equation of the line that passes through the points $(-1, 2)$ and $(7, -2)$.
Give the equation of the line in slope-intercept form.
(b) Find the equation of the line that passes through the point $(-5, 3)$ and is perpendicular to the line $5x-2y=7$.

Q6. Sketch the graphs given by the following equations.
Indicate clearly the location of any **intercepts** with the axes and of any **asymptotes** for the graph.

(a) $y=-2(x-2)^2+2$ (b) $y=\frac{1}{(x+1)^2}+4$

Q7. Solve the following pairs of simultaneous equations to find the coordinates of the points of intersection of the corresponding graphs.

(a) $\begin{cases} x^2+y^2=5 \\ xy=2 \end{cases}$ (b) $\begin{cases} x^2-2xy+y^2=1 \\ x+y=1 \end{cases}$

Q8. Solve the inequalities and, in each case, give the solution set in interval notation.

(a) $\frac{3}{4}-\frac{2}{5}x>1-\frac{1}{2}x$ (b) $|4x-3|\geq 9$ (c) $3x^2-8x>3$ (d) $|x^2-5|\leq 4$ (e) $\frac{2x-1}{x+2}\geq 2$ (f) $\left|\frac{3x+1}{x}\right|<2$

Q9. (a) Given $f(x)=2x^2-3x+5$ and $g(x)=\frac{1}{x-4}$ find $(f \circ g)(x)$ and $(g \circ f)(x)$.

(b) Find two functions f and g such that $(f \circ g)(x)=\frac{1}{\sqrt{x^2+3x}}$.

Q10. The functions f, g and h are defined by $f(x)=\frac{x-2}{x+1}$, $g(x)=4+x^2$ and $h(x)=\sqrt{13-x}$.
(a) Say whether the function g has an inverse and explain why or why not?
(b) Find the inverse function $f^{(-1)}$ of f.
(c) Find $(h \circ g)(x)$ and give the domain of $h \circ g$.

Q11. (a) Given $f(x)=\sqrt{x+1}-2$, find $f^{(-1)}(x)$. Sketch the graphs of f and $f^{(-1)}$. What is the domain of $f^{(-1)}$?
(b) Given $f(x)=x^5+1$, find $f^{(-1)}(x)$. Sketch the graphs of f and $f^{(-1)}$.

Q12. Factor completely: (a) $24x^4+81x$ (b) a^6-64b^6 (c) s^6-7s^3-8 (d) $x^3+x^2-10x+8$
(e) x^4-x^3+8x-8 (f) $x^5+x^4-8x^3-8x^2-9x-9$

Q13. Simplify and factor $x^3 (x-2)^{\left(\frac{5}{3}\right)} - x^4 (x-2)^{\left(-\frac{1}{3}\right)}$, leaving your answer with only positive exponents.

Q14. Find an alternative expression for $\dfrac{x}{1-\sqrt{1-x^2}}$ by rationalizing the denominator.

Q15. Solve for x: (a) $5^{(2x+5)} = \dfrac{1}{125}$ (b) $4^{(5x+1)} = 8^{(2x-1)}$ (c) $7^{(x+1)} = 35$ (Give your answer to 3 significant digits.)

(d) $2\log_2 x - \log_2(3x-8) = 2$ (e) $\log_{10}(x+5) - \log_{10} 3 = \log_{10} 2 - \log_{10} x$.

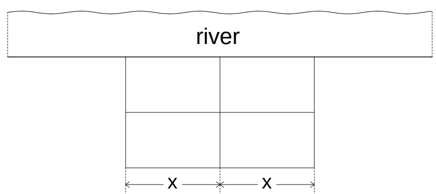
Q16. Evaluate $\log_2 19$ to four decimal places.

Q17. Simplify: (a) $\dfrac{\mathbf{e}^{(3x)} - 1}{\mathbf{e}^{(2x)} - 1}$ (b) $x^2\, \mathbf{e}^{\left(2\ln\left(\frac{1}{\sqrt{x}}\right)\right)} + \ln(\mathbf{e}^{(2x)})$.

Q18. (a) Given $f(x) = \ln(x-1)$, find $f^{(-1)}(x)$. Sketch the graphs of f and $f^{(-1)}$.

(b) Given $f(x) = \mathbf{e}^{(-x+1)} + 2$, find $f^{(-1)}(x)$. Sketch the graphs of f and $f^{(-1)}$.

Q19. A rectangular field of area 1600 square metres next to a river is to be subdivided into four congruent rectangular pens as shown in the diagram. If the length of each pen is x metres, express the length of the fencing used in terms of x.



Q20. If the perimeter of a rectangular flag is 34 in. and the length of a diagonal is 13 in., find the length and height of the flag.

Solutions.

Note: The symbol <=> means "**is equivalent to**" while the symbol => means "**implies**".

Q1. (a) $-\dfrac{7}{24}$ (b) $\dfrac{q}{r} + \dfrac{r}{p} - \dfrac{p}{qr} = \dfrac{p\,q^2 + q\,r^2 - p^2}{p\,q\,r}$

(c) $$\frac{\frac{1}{z}-\frac{1}{y}}{x} - \frac{\frac{1}{x}-\frac{1}{y}}{z} = \frac{\left(\frac{y-z}{yz}\right)}{x} - \frac{\left(\frac{y-x}{xy}\right)}{z} = \frac{y-z}{xyz} - \frac{y-x}{xyz} = \frac{(y-z)-(y-x)}{xyz} = \frac{y-z-y+x}{xyz} = \frac{x-z}{xyz}$$

(d) $$\frac{(125\,a^3\,b^3)^{\left(\frac{2}{3}\right)}}{(25\,a\,b)^{\left(\frac{1}{2}\right)}} = \frac{25\,a^2\,b^2}{5\,a^{\left(\frac{1}{2}\right)}\,b^{\left(\frac{1}{2}\right)}} = 5\,a^{\left(\frac{3}{2}\right)}\,b^{\left(\frac{3}{2}\right)} = 5\,a\sqrt{a}\;b\sqrt{b} = 5\,a\,b\sqrt{a\,b}.$$

Q2. (a) $$(x^{(-1)} + y^{(-1)})^{(-1)} = \frac{1}{\frac{1}{x}+\frac{1}{y}} = \frac{1}{\left(\frac{y+x}{xy}\right)} = \frac{xy}{x+y}$$ (b) $$\frac{(x^3\,y^{(-2)})^6}{(y^{(-5)}\,x^{(-2)})^{(-3)}} = \frac{x^{18}\,y^{(-12)}}{y^{15}\,x^6} = x^{12}\,y^{(-27)} = \frac{x^{12}}{y^{27}}.$$

(c) $$\frac{v^{(-3)}+u^{(-3)}}{v^{(-2)}-u^{(-2)}} = \frac{\frac{1}{v^3}+\frac{1}{u^3}}{\frac{1}{v^2}-\frac{1}{u^2}} = \frac{\left(\frac{u^3+v^3}{u^3\,v^3}\right)}{\left(\frac{u^2-v^2}{u^2\,v^2}\right)} = \left(\frac{u^3+v^3}{u^3\,v^3}\right)\left(\frac{u^2\,v^2}{u^2-v^2}\right) = \frac{(u+v)(u^2-u\,v+v^2)}{u\,v\,(u-v)\,(u+v)} = \frac{u^2-u\,v+v^2}{u\,v\,(u-v)}.$$

Q3. (a) $$\frac{2.58\,.\,10^{12}}{3.14\,.\,10^{(-7)} - 3.52\,.\,10^{(-8)}} = \frac{2.58\,.\,10^{12}}{31.4\,.\,10^{(-8)} - 3.52\,.\,10^{(-8)}} = \frac{2.58\,.\,10^{12}}{(31.4-3.52)\,.\,10^{(-8)}} = \frac{2.58}{31.4-3.52}\,.\,10^{20}$$
$$= \frac{2.58}{27.88}\,.\,10^{20} \simeq 0.09254\,.\,10^{20} \simeq 9.25\,.\,10^{18}.$$

(b) Let the original price be x dollars. Then $\frac{45}{100}x = 108$. Hence $x = \frac{108\,.\,100}{45} = \frac{108\,.\,100}{9\,.\,5} = 12\,.\,20 = 240$.

The original price was $240.

Q4. (a) $\frac{1}{6}x + \frac{1}{3} = 3x - \frac{1}{4}$ <=> $2x + 4 = 36x - 3$ <=> $7 = 34x$ <=> $x = \frac{7}{34}$.

Note: The first step follows on multiplying both sides of the equation by 12, which is the LCM of the denominators 6, 3 and 4.

(b) $2x^2 + 9x - 5 = 0$ <=> $(2x - 1)(x + 5) = 0$ <=> $x = \frac{1}{2}$ or $x = -5$.

(c) $x^2 + 4x - 9 = 0$ <=> $x = \frac{-4 \text{ +/- } \sqrt{16 - (-36)}}{2} = \frac{-4 \text{ +/- } \sqrt{52}}{2} = \frac{-4 \text{ +/- } 2\sqrt{13}}{2} = -2 \pm \sqrt{13}$.

(d) $\frac{x}{\sqrt{16 - x^2}} = 1$ <=> $x = \sqrt{16 - x^2}$ => $x^2 = 16 - x^2$ <=> $2x^2 = 16$ <=> $x^2 = 8$ <=> $x = \pm 2\sqrt{2}$.

Since squaring both sides of an equation may introduce extraneous solutions, the values obtained in the last step should be checked in the original equation. The only solution is $x = 2\sqrt{2}$.

(e) $\frac{3}{x} - \frac{3}{x+2} = 2$ <=> $\frac{(3x+6) - 3x}{x(x+2)} = 2$ <=> $\frac{6}{x(x+2)} = 2$ <=> $6 = 2x(x+2)$ <=> $3 = x(x+2)$

<=> $3 = x^2 + 2x$ <=> $0 = x^2 + 2x - 3$ <=> $(x - 1)(x + 3) = 0$ <=> $x = 1$ or $x = -3$.

(f) $(\sqrt{x} + 2)^3 - 64 = 0$ <=> (f) $(\sqrt{x} + 2)^3 = 64$ <=> $\sqrt{x} + 2 = 4$ <=> $\sqrt{x} = 2$ <=> $x = 4$.

Q5. (a) The line that passes through the points $(-1, 2)$ and $(7, -2)$ has slope

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 2}{7 - (-1)} = \frac{-4}{8} = -\frac{1}{2}.$$

Its equation can be found by using the point-slope form for the equation of a line:

$$y - y_0 = m(x - x_0) \text{ ------- (i),}$$

with $m = \frac{1}{2}$ and (x_0, y_0) taken to be the point $(-1, 2)$.

This gives:

$$y - 2 = -\frac{1}{2}(x - (-1)) \iff y - 2 = -\frac{1}{2}(x + 1) \iff y - 2 = -\frac{1}{2}x - \frac{1}{2} \iff y = -\frac{1}{2}x - \frac{1}{2} + 2 \iff y = -\frac{1}{2}x + \frac{3}{2}.$$

(b) The equation $5x - 2y = 7$ can be written in the form $y = \frac{5}{2}x - \frac{7}{2}$. Hence this line has slope $m = \frac{5}{2}$.

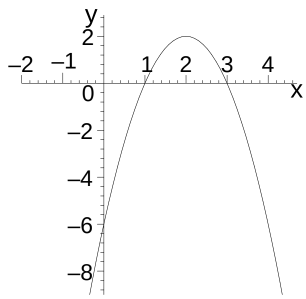
The slope of a perpendicular line is $-\frac{1}{m} = -\frac{2}{5}$. The perpendicular line passing through $(-5, 3)$ has equation:

$$y - 3 = -\frac{2}{5}(x + 5) \iff y - 3 = -\frac{2}{5}x - 2 \iff y = -\frac{2}{5}x + 1.$$

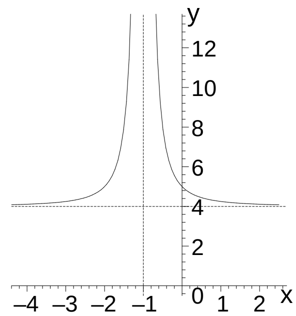
Q6. (a) The graph given by the equation $y = -2(x - 2)^2 + 2$ is obtained by shifting the graph of $y = -2x^2$ a distance of 2 units to the right and 2 units up. The x intercepts are where $x = 1$ and $x = 3$. The y intercept is where $y = -6$.

(b) The graph given by the equation $y = \frac{1}{(x+1)^2} + 4$ is obtained by shifting the graph of $y = \frac{1}{x^2}$ a distance of 1 units to the left and 4 units up. There are no x intercepts. The y intercept is where $y = 5$.

The line $y = 4$ is a horizontal asymptote and the line $x = -1$ is a vertical asymptote.



(a)



(b)

Q7. (a)

$$x^2 + y^2 = 5 \text{ ------- (i)}$$
$$xy = 2 \text{ ------- (ii)}$$

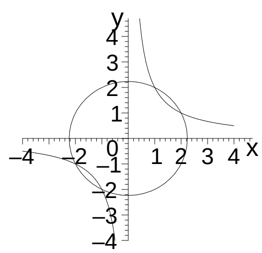
Solving (ii) for y gives

$$y = \frac{2}{x} \text{ ------- (iii)}$$

Substituting for y from (iii) in (i) gives:

$$x^2 + \frac{4}{x^2} = 5 \iff x^4 + 4 = 5x^2 \iff x^4 - 5x^2 + 4 = 0 \iff (x^2 - 4)(x^2 - 1) = 0 \iff x = \pm 2 \text{ or } x = \pm 1.$$

There are 4 solutions: ($x = 1$ and $y = 2$), ($x = -1$ and $y = -2$), ($x = 2$ and $y = 1$), ($x = -2$ and $y = -1$).
The graphs meet at the points $(1, 2)$, $(-1, -2)$, $(2, 1)$ and $(-2, -1)$.



(b)

$$x^2 - 2xy + y^2 = 1 \text{ ------- (i)}$$
$$x + y = 1 \text{ ------- (ii)}$$

Solving (ii) for y gives

$$y = 1 - x \text{ ------- (iii)}$$
$$x^2 - 2xy + y^2 = 1 \iff (x - y)^2 = 1 \iff x - y = \pm 1 \iff y = x \pm 1.$$

The last equation together with (iii) gives:

$$1 - x = x \pm 1 \iff 1 \pm 1 = 2x \iff x = 0 \text{ or } x = 1.$$

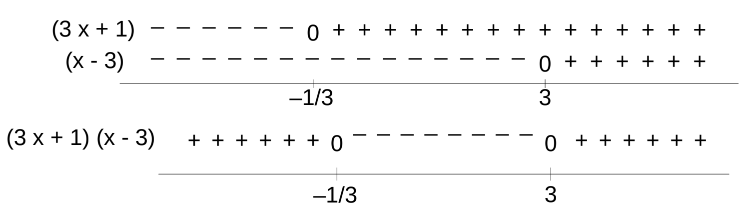
One solution is $x = 0$ and $y = 1$, and another solution is $x = 1, y = 0$. The two graphs meet at the points $(0, 1)$ and $(1, 0)$.
Note: The graph of (i) is the pair of straight lines $y = x + 1$ and $y = x - 1$.

Q8. (a) $\frac{3}{4} - \frac{2}{5}x > 1 - \frac{1}{2}x \iff 15 - 8x > 20 - 10x \iff 2x > 5 \iff x > \frac{5}{2}$. The solution set is: $\left(\frac{5}{2}, \infty\right)$

(b) $|4x - 3| \geq 9 \iff 4x - 3 \leq -9$ or $4x - 3 \geq 9 \iff 4x \leq -6$ or $4x \geq 12 \iff x \leq -\frac{3}{2}$ or $x \geq 3$.

The solution set is: $(-\infty, -\frac{3}{2}] \cup [3, \infty)$.

(c) $3x^2 - 8x > 3 \iff 3x^2 - 8x - 3 > 0 \iff (3x + 1)(x - 3) > 0$.



The last sign chart shows that $(3x + 1)(x - 3) > 0$ exactly when $x < -\frac{1}{3}$ or $x > 3$.

The solution set of the inequality $3x^2 - 8x > 3$ is $\left(-\infty, -\frac{1}{3}\right) \cup (3, \infty)$.

(d) $|x^2 - 5| \le 4 \iff -4 \le x^2 - 5 \le 4 \iff 1 \le x^2 \le 9.$

When $x \ge 0$, $1 \le x^2 \le 9$ is equivalent to $1 \le x \le 3$. When $x < 0$, $1 \le x^2 \le 9$ is equivalent to $-3 \le x \le -1$.

The solution set is: $[-3, -1] \cup [1, 3]$.

(e) $\dfrac{2x-1}{x+2} \ge 2 \iff \dfrac{2x-1}{x+2} - 2 \ge 0 \iff \dfrac{2x-1-2(x+2)}{x+2} \ge 0 \iff \dfrac{5}{x+2} \ge 0 \iff \dfrac{1}{x+2} \ge 0 \iff x+2 \ge 0 \iff x \ge -2.$

The solution set is: $[-2, \infty)$.

(f) $\left|\dfrac{3x+1}{x}\right| < 2 \iff -2 < \dfrac{3x+1}{x} < 2 \iff -2 < 3 + \dfrac{1}{x} < 2 \iff -5 < \dfrac{1}{x} < -1.$

The last (double) inequality implies that x is negative. In general, if a and b are both positive or both negative, $a < b \iff \dfrac{1}{a} > \dfrac{1}{b}$.

Hence $-5 < \dfrac{1}{x} < -1 \iff -1 < x < -\dfrac{1}{5}$. The solution set is: $\left(-1, -\dfrac{1}{5}\right)$

Q9. (a) Given $f(x) = 2x^2 - 3x + 5$ and $g(x) = \dfrac{1}{x-4}$ then

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x-4}\right) = 2\left(\frac{1}{x-4}\right)^2 - 3\left(\frac{1}{x-4}\right) + 5 = \frac{2}{(x-4)^2} - \frac{3}{x-4} + 5.$$

$$(g \circ f)(x) = g(f(x)) = g(2x^2 - 3x + 5) = \frac{1}{(2x^2 - 3x + 5) - 4} = \frac{1}{2x^2 - 3x + 1}.$$

(b) If $h(x) = \dfrac{1}{\sqrt{x^2 + 3x}}$, let $g(x) = x^2 + 3x$ and $f(x) = \dfrac{1}{\sqrt{x}}$. Then $(g \circ f)(x) = h(x)$.

Q10. Given $f(x) = \dfrac{x-2}{x+1}$, $g(x) = 4 + x^2$ and $h(x) = \sqrt{13 - x}$.

(a) The function g does not have an inverse because it is not one-to-one. For example, g associates the two distinct x values $x = -2$ and $x = 2$ with the value $y = 8$. (The graph of g fails the horizontal line test.)

(b) $y = \dfrac{x-2}{x+1} \Rightarrow xy + y = x - 2 \Rightarrow y + 2 = x - xy \Rightarrow y + 2 = x(1-y) \Rightarrow x = \dfrac{y+2}{1-y}.$

Hence $f^{(-1)}(y) = \dfrac{y+2}{1-y}$ so that $f^{(-1)}(x) = \dfrac{x+2}{1-x}$.

(c) $(h \circ g)(x) = h(g(x)) = h(4 + x^2) = \sqrt{13 - (4 + x^2)} = \sqrt{9 - x^2}$.

The domain of $h \circ g$ is $\{ x \mid 9 - x^2 \ge 0 \}$. Now $9 - x^2 \ge 0 \iff x^2 \le 9 \iff -3 \le x \le 3$.

Hence the domain of $h \circ g$ is the interval $[-3, 3]$.

Q11. (a) Given $f(x) = \sqrt{x+1} - 2$, we wish to find $f^{(-1)}(x)$.

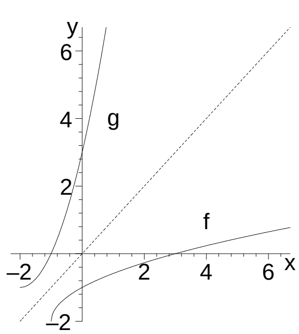
$$y = \sqrt{x+1} - 2 \iff y + 2 = \sqrt{x+1} \iff (y+2)^2 = x + 1 \iff x = (y+2)^2 - 1$$

Hence

$$f^{(-1)}(y) = (y+2)^2 - 1,$$

so that

$$f^{(-1)}(x) = (x+2)^2 - 1 \text{ ------- (i).}$$



In the previous picture $g = f^{(-1)}$.

The domain of $f^{(-1)}$ is the same as the range of f. Hence the **domain** of $f^{(-1)}$ is $[-2, \infty)$.

Note: The formula, (i) should really be qualified with the condition that $x \ge -2$. Thus the complete formula for $f^{(-1)}$ is:

$$f^{(-1)}(x) = (x+2)^2 - 1,\ x \ge -2.$$

(b) Given $f(x) = x^5 + 1$, we wish to find $f^{(-1)}(x)$.

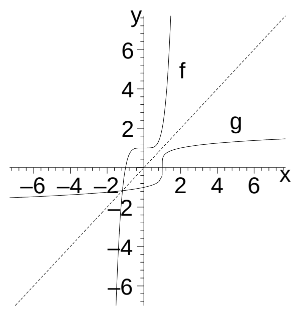
$$y = x^5 + 1 \iff y - 1 = x^5 \iff x = \sqrt[5]{y-1}$$

Hence

$$f^{(-1)}(y) = \sqrt[5]{y-1},$$

so that

$$f^{(-1)}(x) = \sqrt[5]{x-1}, \text{ ------- (i).}$$



In the previous picture $g = f^{(-1)}$.

Q12. (a) $24x^4 + 81x = 3x(8x^3 + 27) = 3x(2x+3)(4x^2 - 6x + 9)$

(b) $a^6 - 64b^6 = (a^3)^2 - (8b^3)^2 = (a^3 - 8b^3)(a^3 + 8b^3) = (a-2b)(a^2 + 2ab + 4b^2)(a+2b)(a^2 - 2ab + 4b^2)$

(c) $s^6 - 7s^3 - 8 = (s^3 - 8)(s^3 + 1) = (s-2)(s^2 + 2s + 4)(s+1)(s^2 - s + 1)$

(d) Let $P(x) = x^3 + x^2 - 10x + 8$. Then $P(1) = 0$, so, by the Factor Theorem, $x - 1$ is a factor of $P(x)$.

$$\begin{array}{r|l} & x^2 + 2x - 8 \\ \hline x - 1 & x^3 + x^2 - 10x + 8 \\ & x^3 - x^2 \\ \hline & 2x^2 - 10x \\ & 2x^2 - 2x \\ \hline & -8x + 8 \\ & -8x + 8 \\ \hline & 0 \end{array}$$

$P(x) = (x-1)(x^2 + 2x - 8) = (x-1)(x+4)(x-2)$

(e) $x^4 - x^3 + 8x - 8 = x^3(x-1) + 8(x-1) = (x^3 + 8)(x-1) = (x+2)(x^2 - 2x + 4)(x-1)$

(f) $x^5 + x^4 - 8x^3 - 8x^2 - 9x - 9 = x^4(x+1) - 8x^2(x+1) - 9(x+1) = (x^4 - 8x^2 - 9)(x+1)$

$$= (x^2 - 9)(x^2 + 1)(x+1) = (x-3)(x+3)(x^2 + 1)(x+1)$$

Q13. $x^3(x-2)^{\left(\frac{5}{3}\right)} - x^4(x-2)^{\left(-\frac{1}{3}\right)} = x^3\left((x-2)^{\left(\frac{5}{3}\right)} - \dfrac{x}{(x-2)^{\left(\frac{1}{3}\right)}}\right) = \dfrac{x^3((x-2)^2 - x)}{(x-2)^{\left(\frac{1}{3}\right)}} = \dfrac{x^3(x^2 - 4x + 4 - x)}{(x-2)^{\left(\frac{1}{3}\right)}}$

$$= \frac{x^3(x^2 - 5x + 4)}{(x-2)^{\left(\frac{1}{3}\right)}} = \frac{x^3(x-1)(x-4)}{(x-2)^{\left(\frac{1}{3}\right)}}$$

Q14 . $\dfrac{x}{1-\sqrt{1-x^2}} = \dfrac{x}{1-\sqrt{1-x^2}} \cdot \dfrac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}} = \dfrac{x(1+\sqrt{1-x^2})}{1-(1-x^2)} = \dfrac{x(1+\sqrt{1-x^2})}{x^2} = \dfrac{1+\sqrt{1-x^2}}{x}$

Q15. (a) $5^{(2x+5)} = \dfrac{1}{125} \Leftrightarrow 5^{(2x+5)} = 5^{(-3)} \Leftrightarrow 2x+5=-3 \Leftrightarrow 2x=-8 \Leftrightarrow x=-4.$

(b) $4^{(5x+1)} = 8^{(2x-1)} \Leftrightarrow (2^2)^{(5x+1)} = (2^3)^{(2x-1)} \Leftrightarrow 2^{(10x+2)} = 2^{(6x-3)} \Leftrightarrow 10x+2 = 6x-3 \Leftrightarrow 4x=-5 \Leftrightarrow x = -\dfrac{5}{4}.$

(c) $7^{(x+1)} = 35 \Leftrightarrow \log_{10}(7^{(x+1)}) = \log_{10} 35 \Leftrightarrow (x+1)\log_{10}(7) = \log_{10} 35 \Leftrightarrow x+1 = \dfrac{\log_{10} 35}{\log_{10} 7}$

$$\Leftrightarrow x = \frac{\log_{10} 35}{\log_{10} 7} - 1 = \frac{\log_{10} 5 + \log_{10} 7}{\log_{10} 7} - 1 = \frac{\log_{10} 5}{\log_{10} 7} \simeq \frac{0.6990}{0.8451} \simeq 0.827.$$

(d) $2\log_2 x - \log_2(3x-8) = 2 \Rightarrow \log_2(x^2) - \log_2(3x-8) = 2 \Rightarrow \log_2\left(\dfrac{x^2}{3x-8}\right) = 2 \Leftrightarrow \dfrac{x^2}{3x-8} = 4 \Leftrightarrow x^2 = 12x - 32$

$\Leftrightarrow x^2 - 12x + 32 = 0 \Leftrightarrow (x-4)(x-8) = 0 \Leftrightarrow x=4$ or $x=8$

$x=4$ and $x=8$ are both solutions. Both of these values check in the original equation.

(e) $\log_{10}(x+5) - \log_{10} 3 = \log_{10} 2 - \log_{10} x \Leftrightarrow \log_{10}\left(\dfrac{x+5}{3}\right) = \log_{10}\left(\dfrac{2}{x}\right) \Rightarrow \dfrac{x+5}{3} = \dfrac{2}{x} \Leftrightarrow x^2+5x=6 \Leftrightarrow x^2+5x-6=0$

$\Leftrightarrow (x+6)(x-1)=0 \Leftrightarrow x=-6$ or $x=1$. Since $\log_{10}(-6)$ does not exist, the only solution is $x=1$.
This value checks in the original equation.

Q16. $\log_2 19 = \dfrac{\log_{10} 19}{\log_{10} 2} \simeq \dfrac{1.278754}{0.3010300} \simeq 4.2479$

Q17. (a) $\dfrac{\mathbf{e}^{(3x)}-1}{\mathbf{e}^{(2x)}-1} = \dfrac{(\mathbf{e}^x-1)(\mathbf{e}^{(2x)}+\mathbf{e}^x+1)}{(\mathbf{e}^x-1)(\mathbf{e}^x+1)} = \dfrac{\mathbf{e}^{(2x)}+\mathbf{e}^x+1}{\mathbf{e}^x+1}.$ **Note**: $\mathbf{e}^{(2x)} = (\mathbf{e}^x)^2$ and $\mathbf{e}^{(3x)} = (\mathbf{e}^x)^3$.

(b) $x^2\,\mathbf{e}^{\left(2\ln\left(\frac{1}{\sqrt{x}}\right)\right)} + \ln(\mathbf{e}^{(2x)}) = x^2\,\mathbf{e}^{\ln\left(\left(\frac{1}{\sqrt{x}}\right)^2\right)} + \ln(\mathbf{e}^{(2x)}) = x^2\,\mathbf{e}^{\ln\left(\frac{1}{x}\right)} + \ln(\mathbf{e}^{(2x)}) = x^2 \cdot \dfrac{1}{x} + 2x = x + 2x = 3x$, where $x > 0$.

Q18. (a) Given $f(x) = \ln(x-1)$, we wish to find $f^{(-1)}(x)$.

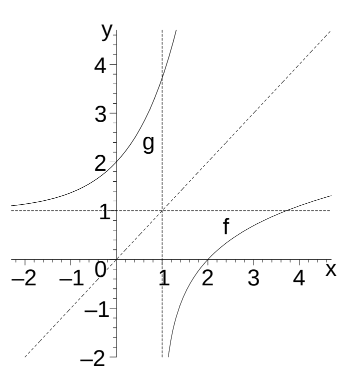
$$y = \ln(x-1) \Leftrightarrow x-1 = \mathbf{e}^y \Leftrightarrow x = \mathbf{e}^y + 1$$

Hence

$$f^{(-1)}(y) = \mathbf{e}^y + 1$$

so that

$$f^{(-1)}(x) = \mathbf{e}^x + 1, \text{ ------- (i).}$$



In the previous picture $g = f^{(-1)}$.

(b) Given $f(x) = \mathbf{e}^{(-x+1)} + 2$, we wish to find $f^{(-1)}(x)$.

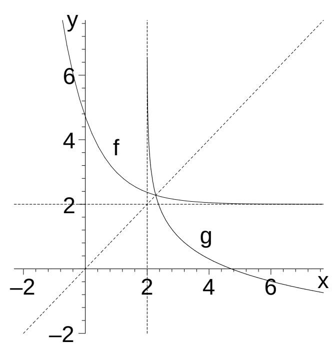
$$y = \mathbf{e}^{(-x+1)} + 2 \iff y - 2 = \mathbf{e}^{(-x+1)} \iff \ln(y-2) = -x + 1 \iff x = 1 - \ln(y-2)$$

Hence

$$f^{(-1)}(y) = 1 - \ln(y-2)$$

so that

$$f^{(-1)}(x) = 1 - \ln(x-2), \text{ ------- (i).}$$

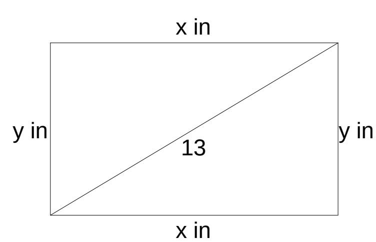


In the previous picture g = $f^{(-1)}$.

Note: $f(x) = \mathbf{e}^{(-x+1)} + 2 = \mathbf{e}^{(-(x-1))} + 2$, so the graph of f is obtained by shifting the graph of $y = \mathbf{e}^{(-x)}$ one unit to the right and 2 units up. The y intercept is $\mathbf{e} + 2 \simeq 4.72$.

Q19. Let the height of each of the four pens be y metres. Since the area of each pen is $\frac{1600}{4} = 400$ square metres, we have $xy = 400$, so that $y = \frac{400}{x}$. Then the length of fencing needed is $4x + 6y = 4x + 6\left(\frac{400}{x}\right) = 4x + \frac{2400}{x}$ metres.

Q20. Let the length and height of the flag be x in. and y in. respectively.



Since the length of a diagonal is 13 in., Pythagoras' Theorem gives

$$x^2 + y^2 = 169 \text{ ------- (i).}$$

Since the perimeter is 34 in., $2x + 2y = 34$, so that, $x + y = 17$.
This gives

$$y = 17 - x \text{ ------- (ii).}$$

Using equation (ii) to substitute for y in (i) gives:

$$x^2 + (17-x)^2 = 169 \iff x^2 + 289 - 34x + x^2 = 169 \iff 2x^2 - 34x + 120 = 0 \iff x^2 - 17x + 60 = 0$$
$$\iff (x-5)(x-12) = 0 \iff x = 5 \text{ or } x = 12.$$

Ans: length = 12 in, height = 5 in. (assuming that the length is greater than the height).