MATH 241
FINAL EXAM FORMULA SHEET

<u>Properties of Transposes</u>: $(A^T)^T = A$ $\quad$ $(A+B)^T = A^T + B^T$
$(cA)^T = c(A^T)$ $\quad$ $(AB)^T = B^T A^T$

<u>Properties of Inverses</u>: $(A^{-1})^{-1} = A$ $\quad$ $(cA)^{-1} = \frac{1}{c}A^{-1} \quad c \neq 0$
$(A^T)^{-1} = (A^{-1})^T$ $\quad$ $(AB)^{-1} = B^{-1}A^{-1}$

<u>Least Squares Line</u>: $A = (X^T X)^{-1} X^T Y$

<u>Cramer's Rule</u>: $x_i = \frac{|A_i|}{|A|} \ for \ i = 1..n$

<u>Equivalent Conditions</u>:
A is invertible
$AX = B$ has a unique solution
A is row-equivalent to I
$|A| \neq 0$
Rank$(A) = n$
A can be written as the product of elementary matrices $E_1^{-1}E_2^{-1} \ldots E_k^{-1}$
The rows and the columns are linearly independent

<u>Axioms of Vector Space</u>:
$\vec{u} + \vec{v} \in V$ $\quad$ $c\vec{u} \in V$
$\vec{u} + \vec{v} = \vec{v} + \vec{u}$ $\quad$ $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$
$\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$ $\quad$ $(c+d)\vec{u} = c\vec{u} + d\vec{u}$
$\vec{u} + \vec{0} = \vec{u}$ $\quad$ $c(d\vec{u}) = (cd)\vec{u}$
$\vec{u} + (-\vec{u}) = \vec{0}$ $\quad$ $1(\vec{u}) = \vec{u}$

<u>Properties of vectors</u>:
$0\vec{v} = \vec{0}$ $\quad$ $c\vec{0} = \vec{0}$
if $c\vec{v} = \vec{0}$ then $c = 0$ or $\vec{v} = \vec{0}$ $\quad$ $(-1)\vec{v} = -\vec{v}$

<u>Test for a Subspace</u>: V is non-empty, closed under addition and closed under scalar multiplication

<u>Coordinates of vectors</u>: $[\vec{v}]_B =$ coordinate matrix of $\vec{v}$ relative to basis B

<u>Transition Matrix</u>: Let $B = \{\vec{v}_1, \ldots, \vec{v}_n\}$ and $B' = \{\vec{u}_1, \ldots, \vec{u}_n\}$, then the transition matrix from B to B' can be found by reducing the matrix

$$\left[[\vec{u}_1]_s \cdots [\vec{u}_n]_s \,\middle|\, [\vec{v}_1]_s \cdots [\vec{v}_n]_s \right]$$

<u>Inner Product</u>:
$\langle \vec{u}, \vec{v} \rangle = \langle \vec{v}, \vec{u} \rangle$ $\quad$ $\langle \vec{u}, \vec{v} + \vec{w} \rangle = \langle \vec{u}, \vec{v} \rangle + \langle \vec{u}, \vec{w} \rangle$
$c\langle \vec{u}, \vec{v} \rangle = \langle c\vec{u}, \vec{v} \rangle$ $\quad$ $\langle \vec{v}, \vec{v} \rangle \geq 0$ and $\langle \vec{v}, \vec{v} \rangle = 0$ iff $\vec{v} = \vec{0}$

<u>Properties</u>:
$\langle \vec{0}, \vec{v} \rangle = \langle \vec{v}, \vec{0} \rangle = 0$ $\quad$ $\langle \vec{u} + \vec{v}, \vec{w} \rangle = \langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle$
$c\langle \vec{u}, \vec{v} \rangle = \langle \vec{u}, c\vec{v} \rangle$

<u>Orthogonal Projection</u>: $proj_{\vec{v}}(\vec{u}) = \frac{\langle \vec{u}, \vec{v} \rangle}{\langle \vec{v}, \vec{v} \rangle}\vec{v}$

<u>Angle between vectors</u>: $cos(\theta) = \frac{\langle \vec{u}, \vec{v} \rangle}{\|\vec{u}\|\|\vec{v}\|}$

<u>Orthonormal Basis</u>: if $B = \{\vec{v}_1, \ldots, \vec{v}_n\}$ then $\langle \vec{v_i}, \vec{v_j} \rangle = 0$ for all $i \neq j$ and $\|\vec{v_i}\| = 1$ for all i.

<u>Orthonormalization</u>: $\vec{w}_i = \vec{v}_i - \frac{\langle \vec{v}_i, \vec{w}_1 \rangle}{\langle \vec{w}_1, \vec{w}_1 \rangle}\vec{w}_1 - \cdots - \frac{\langle \vec{v}_i, \vec{w}_{i-1} \rangle}{\langle \vec{w}_{i-1}, \vec{w}_{i-1} \rangle}\vec{w}_{i-1}$ for i = 2 ... n
then normalize each $\vec{w}_i$ to get $\vec{u}_i$.

<u>Linear Transformation</u>: $T : V \to W$ such that T preserves addition and scalar multiplication.