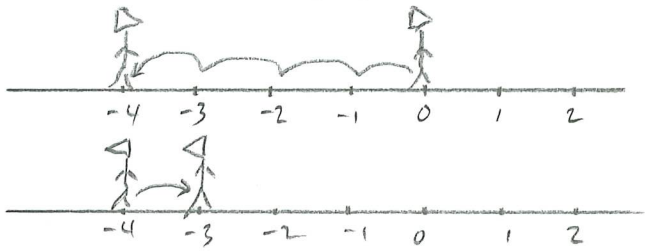
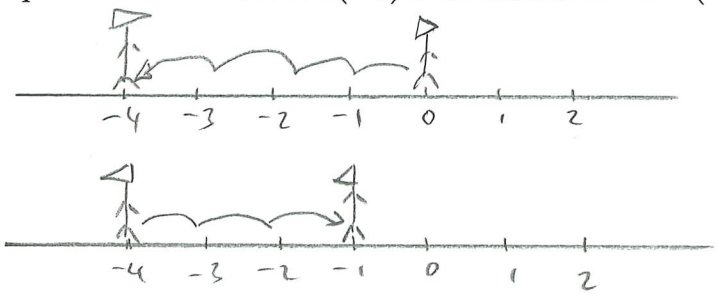
Ex. 5.1A

7. (a) Starting at 0 facing right, move backward 4 steps, turn around, move backward one more steps and the final location (-3) is the answer to $-4-(-1)$.



(b) Starting at 0 facing right, move backward 4 steps, turn around, move backward three more steps and the final location (-1) is the answer to $-4-(-3)$.



14. (a) $-2+(3-10)=-2+(-7)=-9$

(b) $[8-(-5)]-10=13-10=3$

(c) $(-2-7)+10=-9+10=1$

16. (a)

$$\begin{aligned}
3-(2-4x) &= 3+(-(2-4x)) && \text{defn of subtraction}\\
&= 3+(-(2+(-4x))) && \text{defn of subtraction}\\
&= 3+((-2)+(-(-4x))) && \text{negation of sum}\\
&= (3+(-2))+(-(-(4x))) && \text{associative property}\\
&= 1+(-(-(4x))) && \text{addition}\\
&= 1+4x && \text{double negation}
\end{aligned}$$

(b)

$$\begin{aligned}
x-(-x-y) &= x+(-(-x-y)) && \text{defn of subtraction}\\
&= x+(-(-x+(-y))) && \text{defn of subtraction}\\
&= x+((-(-x))+(-(-y))) && \text{negation of sum}\\
&= x+(x+y) && \text{double negation}\\
&= (x+x)+y && \text{associative property}\\
&= 2x+y && \text{addition}
\end{aligned}$$

17. $a-b+c=a-(b-c)$ means

$a-b+c=a+(-(b+(-c)))$

$a-b+c=a+(-b+(-(-c)))$

$a-b+c=a+((-b)+c)$

$a-b+c=a-b+c$

The statement is true for all integers a, b and c.

22. (a) $-x$ is positive if $x \in I^-$

(b) $-x$ is negative if $x \in I^+$

(c) $-x - 1$ is positive mean $-x - 1 > 0$ so $-1 > x$ which means $-x - 1$ is positive for all integers $x < -1$.

(d) $|x| = 2$ when $x = -2$ or $x = 2$.

23. (a) $|x - 6| = 6$ means $x - 6 = -6$ or $x - 6 = 6$, thus $x = 0$ or $x = 12$.

(b) $|x| + 2 = 10$ means $|x| = 8$ so $x = -8$ or $x = 8$.

(c) $|-x| = |x|$ is true for all integers x because the distance from $-x$ to zero will always be the same as the distance from x to zero.

26. (a)
$$\begin{aligned} x + 7 &= 3 \\ x + 7 - 7 &= 3 - 7 \\ x &= -4 \end{aligned}$$

(b)
$$\begin{aligned} -10 + x &= -7 \\ -10 + x + 10 &= -7 + 10 \\ x &= 3 \end{aligned}$$

(c)
$$\begin{aligned} -x &= 5 \\ -x(-1) &= 5(-1) \\ x &= -5 \end{aligned}$$

Ex. 5.1B

16. (a)
$$\begin{aligned} 4x - 2 - 3x &= 4x + (-2) + (-3x) && \text{defn of subtraction} \\ &= 4x + (-3x) + (-2) && \text{commutative property} \\ &= (4 + (-3))x + (-2) && \text{distributive property} \\ &= 1x + (-2) && \text{addition} \\ &= x - 2 && \text{defn of subtraction} \end{aligned}$$

(b)
$$\begin{aligned} 4x - (2 - 3x) &= 4x + (-(2 - 3x)) && \text{defn of subtraction} \\ &= 4x + (-(2 + (-3x))) && \text{defn of subtraction} \\ &= 4x + ((-2) + (-(-(3x)))) && \text{negation of sum} \\ &= 4x + ((-2) + 3x) && \text{double negation} \\ &= 4x + (3x + (-2)) && \text{commutative property} \\ &= (4x + 3x) + (-2) && \text{associative property} \\ &= 7x + (-2) && \text{addition} \\ &= 7x - 2 && \text{defn of subtraction} \end{aligned}$$

20. There are $52 - (-28) = 80$ terms in the arithmetic sequence $-27, -26, \ldots, 52$.

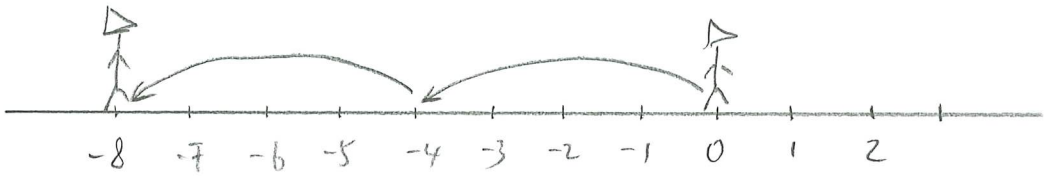
26. (a) $-x+5=7$
$-x+5-5=7-5$
$-x=2$
$(-x)(-1)=(2)(-1)$
$x=-2$

(b) $1-x=-13$
$1-x-1=-13-1$
$-x=-14$
$x=14$

(c) $-x-8=-9$
$-x-8+8=-9+8$
$-x=-1$
$x=1$

Ex. 5.2A

3. Starting at 0 facing right, make two jumps of 4 backward steps each and the final location (-8) is the answer to $2\cdot(-4)$.



7. (a) $((-10)\div(-2))(-2)=-10$ because $(-10)\div(-2)$ is a number when multiplied by -2 gives -10.
(b) $(-10\cdot 5)\div 5=-10$ because $-10\cdot 5\div 5$ is a number when multiplied by 5 gives $-10\cdot 5$.
(c) $-8\div(-8+8)=-8\div 0=$ undefined
(d) $(-6+6)\div(-2+2)=0\div 0=$ undefined
(e) $|-24|\div[4(9-15)]=24\div(4\times(-6))=24\div(-24)=-1$

13. (a) $(-2)^3=-8$
(b) $(-2)^4=16$
(c) $(-10)^5\div(-10)^2=(-10)^3=-1000$
(d) $(-3)^5\div(-3)=(-3)^4=81$
(e) $(-1)^{50}=1$
(f) $(-1)^{151}=-1$
(g) $-2+3\cdot 5-1=-2+15-1=12$
(h) $10-3\cdot 7-4(-2)+3=10-21+8+3=0$

17. (a) $(-3)(4+5) = (4+5)(-3)$ commutative property

(b) $(-4)(-7) \in I$ closure property

(c) $5(4(-3)) = (5 \cdot 4)(-3)$ associative property

(d) $(-9)(5+(-8)) = (-9)(5) + (-9)(-8)$ distributive property

19. (a) $(-2)(x-y) = (-2)x - (-2)y = -2x + 2y$

(b) $x(x-y) = x \cdot x - x \cdot y = x^2 - xy$

(c) $-x(x-y) = (-x)x - (-x)y = -x^2 - (-xy) = -x^2 + xy$

(d) $-2(x+y-z) = (-2)x + (-2)y - (-2)z = -2x - 2y + 2z$

20. (f) $-3x - 8 = 7$

$-3x - 8 + 8 = 7 + 8$

$-3x = 15$

$x = -5$

(g) $-2(5x-3) = 26$

$5x - 3 = 26 \div (-2)$

$5x - 3 + 3 = -13 + 3$

$5x = -10$

$x = -2$

(h) $3x - x - 2x = 3$

$0 = 3$ which is impossible, therefore no solutions.

(i) $-2(5x-6) - 30 = -x$

$-10x + 12 - 30 = -x$

$-10x - 18 + 10x = -x + 10x$

$-18 = 9x$

$x = -2$

22. (e) $abc + ab - a = a \cdot bc + a \cdot b - a \cdot 1 = a(bc + b - 1)$

(f) $16 - a^2 = 4^2 - a^2 = (4+a)(4-a)$

(g) $4x^2 - 25y^2 = (2x)^2 - (5y)^2 = (2x+5y)(2x-5y)$

Ex. 5.2B

7. (a) $(a \div b)b = a$ because $a \div b$ is a number when multiplied by b gives a.
   (b) $(ab) \div b = a$ because it is a number when multiplied by b gives ab.
   (c) $(-8+8) \div 8 = 0 \div 8 = 0$ because $8 \times 0 = 0$.
   (d) $(-23-(-7)) \div 4 = -16 \div 4 = -4$ because $4 \times (-4) = -16$.
   (e) $|-28| \div (2|-7|) = 28 \div 14 = 2$ because $2 \times 14 = 28$

13. (a) $10-3-12 = 7-12 = -5$
    (b) $10-(3-12) = 10-(-9) = 19$
    (c) $(-3)^2 = 9$
    (d) $-3^2 = -9$
    (e) $-5^2+3(-2)^2 = -25+12 = -13$
    (f) $-2^3 = -8$
    (g) $(-2)^5 = -32$
    (h) $-2^4 = -16$

19. (a) $-x(x-y-3) = (-x)x-(-x)y-(-x)3 = -x^2+xy+3x$
    (b) $(-5-x)(5+x) = -25-5x-5x-x^2 = -25-10x-x^2$
    (c) $(x-y-1)(x+y+1) = (x-y-1)x+(x-y-1)y+(x-y-1)1$
    $= x^2-xy-x+xy-y^2-y+x-y-1$
    $= x^2-y^2-2y-1$

22. (e) $(a+b)(c+1)-(a+b) = (a+b)(c+1)-(a+b)\cdot 1$
    $= (a+b)(c+1-1)$
    $= (a+b)\cdot c$
    (f) $x^2-9y^2 = x^2-(3y)^2 = (x+3y)(x-3y)$
    (g) $(x^2-y^2)+(x+y) = (x+y)(x-y)+(x+y)\cdot 1$
    $= (x+y)(x-y+1)$