Ex. 4.1A

3. (a) $2|4,201,012$ because $2|2$ the unit digit.
   $3\nmid 4,201,012$ because $3\nmid 10$ the sum of all the digits.
   $4|4,201,012$ because $4|12$ the number formed by the last two digits.
   $5\nmid 4,201,012$ because the last digit is not 5 or 0.
   $6\nmid 4,201,012$ because $2|4,201,012$ but $3\nmid 4,201,012$.
   $8\nmid 4,201,012$ because $8\nmid 012$ the number formed by the last three digits.
   $9\nmid 4,201,012$ because $9\nmid 10$ the sum of all the digits.
   $10\nmid 4,201,012$ because the last digit is not zero.
   $11\nmid 4,201,012$ because $(2+1+1)-(4+0+0+2)=2$ which is not divisible by 11.

6. (a) Find the largest x such that $7+4+x$ is divisible by 3. The answer is $x=7$.
   (b) Find the largest x such that $8+3+x+4+5$ is divisible by 9. The answer is $x=7$.

9. (b) "If $b|a$ then $(b+c)|(a+c)$." is only true sometimes. For example, $1|3$ and $(1+1)|(3+1)$, however, $2|4$ does not imply $(2+1)|(4+1)$.
   (c) "If $b|a$ then $b^2|a^3$." is always true because if $b|a$ then there exists integer k such that $a=kb$ so $a^3=(kb)^3=k^3b^3=(k^3b)b^2$. Since k^3b is an integer, a^3 is an integer multiple of b^2 so $b^2|a^3$.
   (d) "If $b|a$ then $b|(a+b)$." is always true because if $b|a$ then there exists integer k such that $a=kb$. This means that $(a+b)=(kb+b)=(k+1)b$, since $k+1$ is an integer, $b|(a+b)$.

17. Let the digits for n be $...a_4a_3a_2a_1a_0$, then

$$n = ... + 10000a_4 + 1000a_3 + 100a_2 + 10a_1 + a_0$$
$$= 16(... + 625a_4) + (1000a_3 + 100a_2 + 10a_1 + a_0)$$

therefore, $16|n$ if and only if 16 divides the number formed by the last 4 digits.

Ex. 4.1B

8. (a) $12 \nmid 24013$ because $4 \nmid 24013$.
   (b) $12 | 24036$ because $4 | 24036$ and $3 | 24036$.

16. If $28|n$ then n is a multiple of 28 which is also a multiple of 14, 7, 4, 2 and 1.

17. Let the digits for n be $...a_4a_3a_2a_1a_0$, then

$$n = ... + 10000a_4 + 1000a_3 + 100a_2 + 10a_1 + a_0$$
$$= 25(... + 400a_4 + 40a_3 + 4a_2) + (10a_1 + a_0)$$

therefore, $25|n$ if and only if 25 divides the number formed by the last 2 digits.

20. Let the $n = a_3a_2a_1a_0$ be a four digit number , then

$$n = 1000a_3 + 100a_2 + 10a_1 + a_0$$
$$= (999+1)a_3 + (99+1)a_2 + (9+1)a_1 + a_0$$
$$= (999a_3 + 99a_2 + 9a_1) + (a_3 + a_2 + a_1 + a_0)$$
$$= 9(111a_3 + 11a_2 + a_1) + (a_3 + a_2 + a_1 + a_0)$$

since $(111a_3 + 11a_2 + a_1)$ is an integer, $9|n$ if and only if $(a_3 + a_2 + a_1 + a_0)$ is also a multiple of 9, thus $9|n$ if and only if 9 divides the sum of all 4 digits.