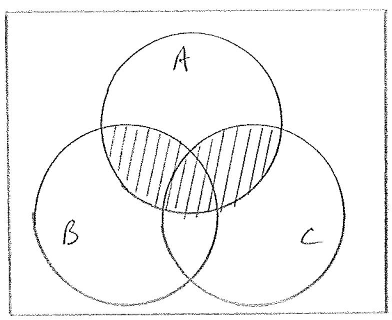
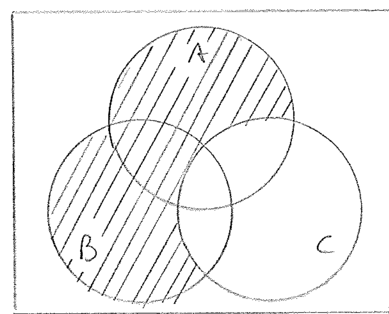
Ex. 2.3A

3. (a) $A \cup \emptyset = A$ is true.
   (b) $A - B = B - A$ is false.
   counterexample: $A = \{1,2,3\},\ B = \{2,3,4\},\ A - B = \{1\}$ but $B - A = \{4\}$
   (c) $\overline{A \cap B} = \overline{A} \cap \overline{B}$ is false.
   counterexample: $U = \{1,2,3,4,\},\ A = \{1,3\},\ B = \{3,4\}$
   $\overline{A \cap B} = \overline{\{3\}} = \{1,2,4\}$ but $\overline{A} \cap \overline{B} = \{2,4\} \cap \{1,2\} = \{2\}$
   (d) $(A \cup B) - A = B$ is false.
   counterexample: $A = \{1,2,3\},\ B = \{3,4\}$
   $(A \cup B) - A = \{1,2,3,4\} - \{1,2,3\} = \{4\} \neq B$
   (e) $(A - B) \cup A = (A - B) \cup (B - A)$ is false.
   counterexample: $A = \{1,2,3\},\ B = \{3,4\}$
   $(A - B) \cup A = \{1,2\} \cup \{1,2,3\} = \{1,2,3\}$
   $(A - B) \cup (B - A) = \{1,2\} \cup \{4\} = \{1,2,4\}$

5. (a) $(A \cap B) \cup (A \cap C)$



(b) $(A \cup B) \cap \overline{C}$

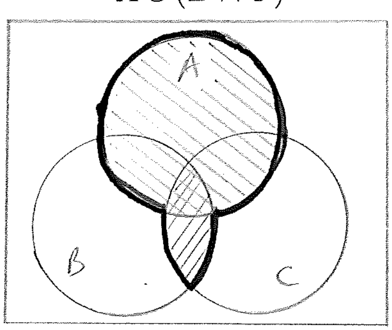


7. (a) If $A \cap B = \emptyset$ then $A - B = A$
   (b) If $B = \emptyset$ then $A - B = A$
   (a) If $B = U$ then $A - B = \emptyset$

9. (a) $B - A$ or $\overline{A} \cap B$
   (b) $\overline{A \cup B}$ or $\overline{A} \cap \overline{B}$
   (c) $(A \cap B) - C$ or $A \cap B \cap \overline{C}$

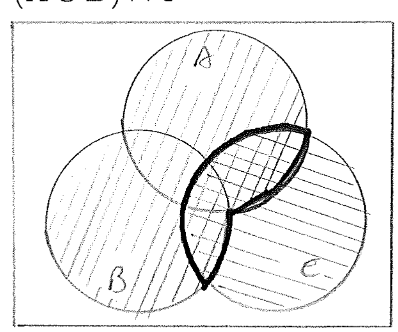
11. (a) $A \cup (B \cap C) \neq (A \cup B) \cap C$ because the two Venn diagrams are not the same:

$A \cup (B \cap C)$



$A \cup (B \cap C)$ includes all shaded region

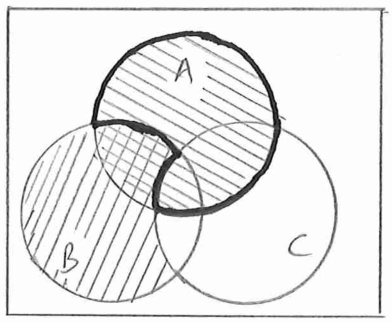
$(A \cup B) \cap C$



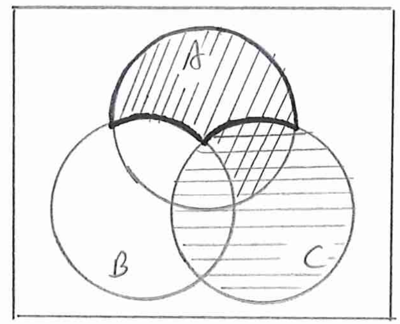
$(A \cup B) \cap C$ is the region shaded by both sets

(b) $A-(B-C) \neq (A-B)-C$ because the two Venn diagrams are not the same:

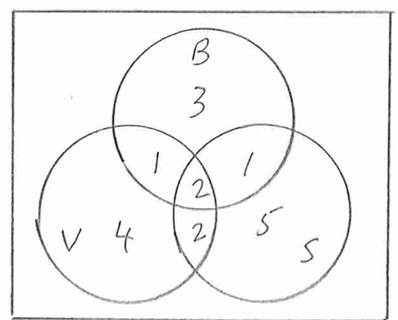
$A-(B-C)$



$(A-B)-C$



18. Let B be the set of basketball players
V be the set of volleyball players
S be the set of soccer players



then
$n(B)=7$
$n(V)=9$
$n(S)=10$
$n(B \cap V \cap \overline{S})=1$
$n(B \cap S \cap \overline{V})=1$
$n(V \cap S \cap \overline{B})=2$
$n(B \cap V \cap S)=2$
we need to find $n(B \cup V \cup S)$.
From the Venn diagram, $n(B \cup V \cup S)=3+1+2+1+4+2+5=18$

27. $A=\{x,\ y\}$ and $B=\{a,\ b,\ c\}$
(a) $A \times B=\{(x,a),\ (x,b),\ (x,c),\ (y,a),\ (y,b),\ (y,c)\}$
(b) $B \times A=\{(a,x),\ (a,y),\ (b,x),\ (b,y),\ (c,x),\ (c,y)\}$
(c) $A \times B \neq B \times A$

28. (a) If $C \times D=\{(a,b),\ (a,c),\ (a,d),\ (a,e)\}$ then $C=\{a\}$ and $D=\{b,c,d,e\}$.
(b) If $C \times D=\{(1,1),\ (1,2),\ (1,3),\ (2,1),\ (2,2),\ (2,3)\}$ then $C=\{1,2\}$ and $D=\{1,2,3\}$.
(c) If $C \times D=\{(0,1),\ (0,0),\ (1,1),\ (1,0)\}$ then $C=\{0,1\}$ and $D=\{0,1\}$.

Ex. 2.3B

8. (a) $A \cap C$
(b) $(A \cup B) \cap C$ or $(A \cap C) \cup (B \cap C)$
(c) $(B \cup C)-A$ *different in first edition

12. (a) If $n(A \cup B) = 23$, $n(A \cap B) = 9$ and $n(B) = 12$ then

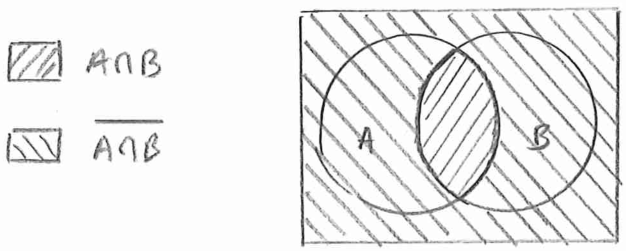
$$23 = n(A) + 12 - 9$$
$$n(A) = 20$$

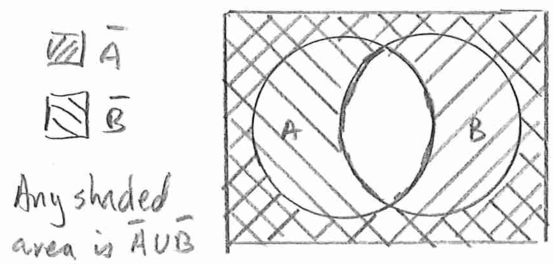
(b) If $n(A) = 9$, $n(B) = 13$ and $n(A \cap B) = 5$ then

$$n(A \cup B) = 9 + 13 - 5$$
$$n(A \cup B) = 17$$

13. (a) $\overline{A \cap B} = \overline{A} \cup \overline{B}$ because both sets cover the same region in the Venn diagram as shown below:

$\overline{A \cap B}$ $\qquad\qquad\qquad\qquad$ $\overline{A} \cup \overline{B}$





(b) Let $U = \{a,\ b,\ c,\ d,\ e,\ f,\ g\}$, $A = \{a,\ b,\ c,\ d\}$, $B = \{b,\ c,\ e.\ f\}$ then

$\overline{A} = \{e,\ f,\ g\}$

$\overline{B} = \{a,\ d,\ g\}$ and $A \cap B = \{b,\ c\}$.

Therefore,

$\overline{A \cap B} = \{a,\ d,\ e,\ f,\ g\}$ and $\overline{A} \cup \overline{B} = \{a,\ d,\ e,\ f,\ g\}$

so $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

17. (a) Regions (c), (f), (g). (k)

(b) Regions (b), (c), (d), (e), (f), (g).

(c) Regions (b), (e).

(d) Region (a) consists of all students who took algebra only and not the other two.

(e) Region (f) consists of all students who took biology and chemistry but not algebra.

(f) $(B \cap C) - A$

(g) $C - B$

(h) $C - (A \cup B)$

MC. 2.3

20. Total no. of combinations $=$ no. of pairs of slacks $\times$ no. of shirts $\times$ no. of sweaters

$$= 3 \times 4 \times 5$$
$$= 60$$