MC. 8.1

3. Let x be the original number and y be the answer, then

$$\frac{4x+16}{2} - 7 = y$$

Solving the equation for x, we get

$$\frac{4x+16}{2} = y + 7$$
$$4x + 16 = 2(y+7)$$
$$4x = 2(y+7) - 16$$
$$4x = 2y + 14 - 16$$
$$4x = 2y - 2$$
$$x = \frac{2(y-1)}{4}$$
$$x = \frac{y-1}{2}$$

Ans: Thus, to get the original number, take the answer, subtract 1 and divide by 2.

Ex. 8.2A

5. Let x be the number of student tickets sold, then

	student	*non-student*	*total*
no. of students	x	$812 - x$	812
revenue	$2x$	$3(812 - x)$	1912

Since the total revenue is \$1912, we must have

$$2x + 3(812 - x) = 1912$$
$$2x + 2436 - 3x = 1912$$
$$2436 - x = 1912$$
$$-x = -524$$
$$x = 524$$

Ans: 524 student tickets were sold

6. Let x be the amount received by the youngest, then

oldest	*middle*	*youngest*	*total*
$3x$	$x + 14000$	x	486000

Since the total is \$486000, we must have

$$3x + (x + 14000) + x = 486000$$
$$5x + 14000 = 486000$$
$$5x = 472000$$
$$x = 94400$$

Ans: The oldest child received \$283,200, the middle child received \$108,400 and the youngest received \$94,400.

11. Let a be the shorter side of the field then the longer side would be $b = 2a$
Since the total amount of fencing is 1200 yard, we have

$$a + a + b = 1200$$

replacing b with $2a$ we get

$$a + a + 2a = 1200$$
$$4a = 1200$$
$$a = 300$$

Ans: The dimension of the rectangular field is 300 yd $\times$ 600 yd.

12. Let x be the smallest of the three consecutive terms in the arithmetic sequence, then

smallest	*middle*	*largest*	*sum of all three*
x	$x+3$	$x+6$	903

Since the sum of all three is 903, we must have

$$x + (x+3) + (x+6) = 903$$
$$3x + 9 = 903$$
$$3x = 894$$
$$x = 298$$

Ans: The three consecutive terms are 298, 301 and 304.

Ex. 2.2A

1. a. $\{x \mid x\, is\, a\, letter\, in\, the\, word\, ASSESSMENT\} = \{a,\ s,\ e,\ m,\ n,\ t\}$
   b. $\{x \mid x\, is\, a\, natural\, number\, greater\, than\, 20\} = \{21,\ 22,\ 23,\ 24,\ \ldots\}$

2. a. $P = \{p,\ q,\ r,\ s\}$
   b. $\{1,\ 2\} \subset \{1,\ 2,\ 3\}$
   c. $\{0,\ 1\} \not\subseteq \{1,\ 2,\ 3\}$

3. a. $\{1,\ 2,\ 3,\ 4,\ 5\}$ and $\{m,\ n,\ o,\ p,\ q\}$ can be put into a one-to-one correspondence.
   b. $\{a,\ b,\ c,\ d,\ \ldots,\ m\}$ and $\{1,\ 2,\ 3,\ 4,\ \ldots,\ 13\}$ can be put into a one-to-one correspondence.
   c. $\{x \mid x\, is\, a\, letter\, in\, the\, word\, mathematics\}$ and $\{1,\ 2,\ 3,\ \ldots,\ 11\}$ cannot be put into a one-to-one correspondence because the first set only has 8 elements in it.

5. The number of one-to-one correspondences between $\{x, y, z, u, v\}$ and $\{1, 2, 3, 4, 5\}$
   a. is $1 \times 4 \times 3 \times 2 \times 1 = 24$ if x must correspond to 5.
   b. is $1 \times 1 \times 3 \times 2 \times 1 = 6$ if x must correspond to 5 and y to 1.
   c. is $3 \times 2 \times 1 \times 2 \times 1 = 12$ if $x,\ y,\ z$ must correspond to odd numbers.

7. a. $n(\{201,\ 202,\ 203,\ \ldots,\ 1100\}) = 1100 - 200 = 900$
   b. $n(\{1,\ 3,\ 5,\ \ldots,\ 101\}) = \frac{102}{2} = 51$
   c. $n(\{1,\ 2,\ 4,\ 8,\ 16,\ \ldots,\ 1024\}) = n(\{2^0,\ 2^1,\ 2^2,\ 2^3,\ \ldots,\ 2^{10}\}) = 11$
   d. $n(\{x \mid x = k^3,\ k = 1,\ 2,\ 3,\ \ldots,\ 100\}) = 100$

11. If $A = \{a, b, c, d, e\}$,
    a. A has $2^5 = 32$ subsets.
    b. A has $32 - 1 = 31$ proper subsets.
    c. Number of subsets containing a and e is $2^3 = 8$.

14. a. $0 \notin \emptyset$
    b. $1024 \in \{x \mid x = 2^n \text{ and } n \in \mathbb{N}\}$ because $1024 = 2^{10}$
    c. $3002 \in \{x \mid x = 3n - 1 \text{ and } n \in \mathbb{N}\}$ because $3002 = 3(1001) - 1$
    d. $\{1\} \notin \{1, 2\}$

15. a. $0 \not\subseteq \emptyset$
    b. $1024 \not\subseteq \{x \mid x = 2^n \text{ and } n \in \mathbb{N}\}$
    c. $3002 \not\subseteq \{x \mid x = 3n - 1 \text{ and } n \in \mathbb{N}\}$
    d. $\{1\} \subseteq \{1, 2\}$

16. a. Yes, if $A = B$ then $A \subseteq B$.
    b. No, if $A \subseteq B$, it is not necessary that $A \subset B$, for example $A = \{1, 2\}$, $B = \{1, 2\}$.
    c. Yes, if $A \subset B$ then $A \subseteq B$.
    d. No, if $A \subseteq B$, it is not necessary that $A = B$, for example $A = \{1\}$, $B = \{1, 2\}$.

Extra question

no. of ways to pick the first digit $\times$ no. of ways to pick the second digit
$= 9 \times 9$
$= 81$