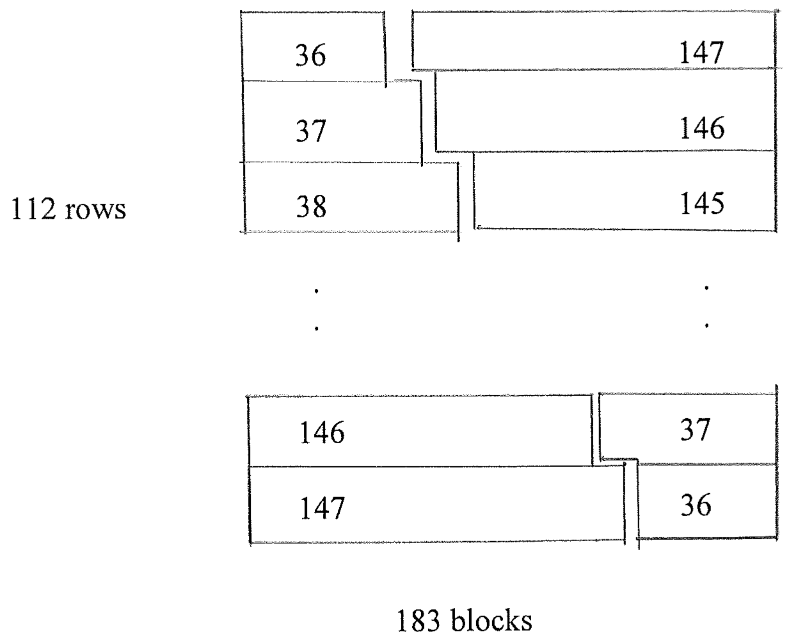
Ex. 1.1A

3. Plan : - draw rows of 36, 37, 38, ... , 147 blocks to make a staircase.
   - make a copy of the staircase and turn it 180°.
   - fit the two staircases together to form a rectangle.
   - count the number of blocks in the rectangle.
   - divide it by two to find the answer.

Carrying out the plan :



Total number of blocks $= 112 \times 183 = 20,496$

Answer : $36 + 37 + 38 + \cdots + 146 + 147 = \frac{20496}{2} = 10,248.$

4.b Plan : - write all combinations of 1's, 2's and 6's that add up to 12 with decreasing number of 1's and 2's.

Carrying out the plan :

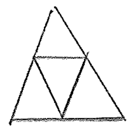
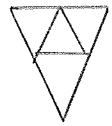
no. of 1's	12	10	8	6	6	4	4	2	2	0	0	0
no. of 2's	0	1	2	3	0	4	1	5	2	6	3	0
no. of 6's	0	0	0	0	1	0	1	0	1	0	1	2

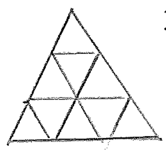
Answer : There are 12 ways to come up with a dozen using only 1's, 2's and 6's.

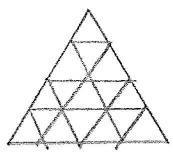
6. Plan : - list all possible types of triangles
- count the number of triangles of each type
- add them up to find the total

Carrying out the plan :

triangles of size 1  10  6

triangles of size 2  6  1

triangles of size 3  3

triangles of size 4  1

Answer : There are 27 triangles in total.

12. Plan : - since the last 3 digits must add up to 20, find the second last digit first, then work our way back to the first digit from right to left.

Carrying out the plan :

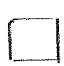
$13^{th}\ digit = 20 - 4 - 7 = 9$
$11^{th}\ digit = 20 - 9 - 7 = 4$
$10^{th}\ digit = 20 - 7 - 4 = 9$
$9^{th}\ digit = 20 - 4 - 9 = 7$
$8^{th}\ digit = 20 - 9 - 7 = 4$

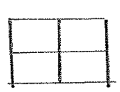
Answer : The pattern of 4, 9, 7 keeps repeating, so the first digit must be 9 ie. $A = 9$.

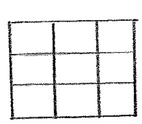
Ex. 1.1B

6. Plan : - list all possible types of squares
- count the number of squares of each type
- add them up to find the total

Carrying out the plan :

squares of size 1  13

squares of size 2  4

squares of size 3  1

Answer : There are 18 squares in total.

10. Plan : - find the sum of all nine numbers, divide it by 3 to find the row sum
- list of possible triples that add to the required sum
- use the list to find the position of each number

Carrying out the plan :

$3+5+7+9+11+13+15+17+19=99$ so row sum is 33.

Triples that add up to 33 are : $3+11+19$, $3+13+17$, $5+9+19$, $5+11+17$, $5+13+15$, $7+9+17$, $7+11+15$, $9+11+13$

Since 11 appears in four different triples, it must go in the middle. 5, 9, 13, 17 all appear in 3 triples, they must be the corners.

Answer :

5	19	9
15	11	7
13	3	17

MC. 1.1

10. It is impossible to have a 3×3 magic square with numbers 1, 3, 4, 5, 6, 7, 8, 9, 10 because the sum of the numbers is 53 which is not divisible by 3 so we cannot have three rows all adding up to the same sum.

Ex. 1.2A

2. a. 1, 5, 9, 13, ***17, 21, 25*** is an arithmetic sequence with $a_1 = 1$ and $d = 4$.
   b. 70, 120, 170, ***220, 270, 320*** is an arithmetic sequence with $a_1 = 70$ and $d = 50$.
   c. 1, 3, 9, ***27, 81, 243*** is a geometric sequence with $a_1 = 1$ and $r = 3$.
   d. 10, 10^3, 10^5, 10^7, $\boldsymbol{10^9, 10^{11}, 10^{13}}$ is a geometric sequence with $a_1 = 10$ and $r = 10^2$.
   e. $193 + 7 \cdot 2^{30}$, $193 + 8 \cdot 2^{30}$, $193 + 9 \cdot 2^{30}$, $\mathbf{193 + 10 \cdot 2^{30}}$, $\mathbf{193 + 11 \cdot 2^{30}}$, $\mathbf{193 + 12 \cdot 2^{30}}$ is an arithmetic sequence with $a_1 = 193 + 7 \cdot 2^{30}$ and $d = 2^{30}$.

3. a. $a_{100} = 1 + (100-1) \times 4 = 397$ $\qquad a_n = 1 + (n-1) \times 4$
   b. $a_{100} = 70 + (100-1) \times 50 = 5020$ $\qquad a_n = 70 + (n-1) \times 50$
   c. $a_{100} = 1 \cdot 3^{100-1} = 1 \cdot 3^{99}$ $\qquad a_n = 1 \cdot 3^{n-1}$
   d. $a_{100} = 10 \cdot (10^2)^{100-1} = 10 \cdot 10^{2\times 99} = 10^{199}$ $\qquad a_n = 10 \cdot (10^2)^{n-1} = 10 \cdot 10^{2n-2} = 10^{2n-1}$
   e. $a_{100} = 193 + 7 \cdot 2^{30} + (100-1) \times 2^{30} = 193 + 7 \cdot 2^{30} + 99 \cdot 2^{30} = 193 + 106 \cdot 2^{30}$ $\qquad a_n = 193 + 7 \cdot 2^{30} + (n-1) \cdot 2^{30} = 193 + (7+n-1) \cdot 2^{30} = 193 + (6+n) \cdot 2^{30}$

6. a. The 10^{th} windmill will have $5 + (10-1) \times 4 = 41$ squares.
   b. The n^{th} windmill will have $5 + (n-1) \times 4$ squares.
   c. It will take $16 + (n-1) \times 12$ matchsticks to build the n^{th} windmill.

12. a. $51, 52, 53, 54, \ldots, 251$ has $251 - 50 = 201$ terms.
b. $1, 2, 2^2, 2^3, \ldots, 2^{60}$ has 61 terms (exponents 0 to 60).
c. $10, 20, 30, 40, \ldots, 2000$ has 200 terms (from one 10 to 200 10's).
d. $1, 2, 4, 8, 16, 32, \ldots, 1024$ has 11 terms (exponents 0 to 10).

15. a. $a_6 = \; = 6^2 + 5 = 41$
b. $a_n = n^2 + (n-1)$
c. Observe that $30^2 = 900$, $40^2 = 1600$ and $35^2 = 1225$ which is the largest square less than 1259. Furthermore, $35^2 + 34$ happens to be 1259, thus $a_{35} = 1259$ or the 35^{th} figure has 1259 squares.

Ex. 1.2B

7. a. The first figure has 1 triangle.
The second figure has $1 + 3$ triangles.
The third figure has $1 + 3 + 5$ triangles.
The 100^{th} figure will have $1 + 3 + 5 + 7 + \ldots + (1 + 99 \times 2)$ triangles. We need to find the sum of $1 + 3 + 5 + \ldots + 199$. Using the technique learned in class, $1 + 3 + 5 + \ldots + 199 = \frac{200 \times 100}{2} = 10000$.
b. For the n^{th} figure, the number of triangles $= 1 + 3 + 5 + \ldots + (1 + (n-1)2)$

$$= \frac{n \times (2n - 1 + 1)}{2} = \frac{2n^2}{2} = n^2$$

Extra question

Given $a_n = 3a_{n-1} - 2a_{n-2}$ with $a_1 = 3$ and $a_2 = 6$.

$a_3 = 3a_2 - 2a_1 = 3(6) - 2(3) = 12$
$a_4 = 3a_3 - 2a_2 = 3(12) - 2(6) = 24$
$a_5 = 3a_4 - 2a_3 = 3(24) - 2(12) = 48$