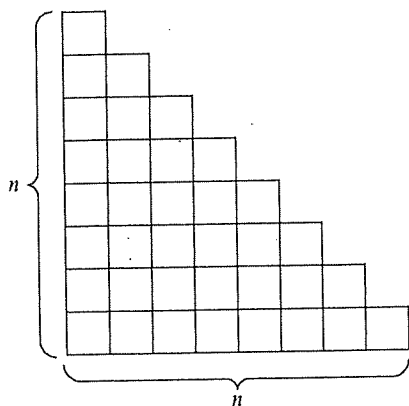
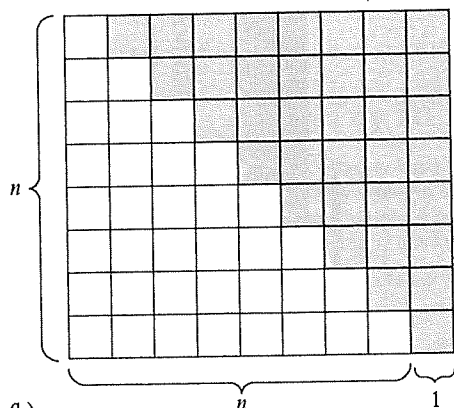


Assessment 1-1A

- Use the approach in Gauss's Problem to find the following sums of arithmetic sequences.
 - $1 + 2 + 3 + 4 + \dots + 99$
 - $1 + 3 + 5 + 7 + \dots + 1001$
 - $3 + 6 + 9 + 12 + \dots + 300$
 - $4 + 8 + 12 + 16 + \dots + 400$
- Use the ideas in drawings (a) and (b) to find the solution to Gauss's Problem for the sum $1 + 2 + 3 + \dots + n$. Explain your reasoning.



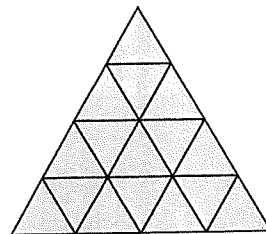
(a)



(b)

- Find the sum $36 + 37 + 38 + 39 + \dots + 146 + 147$.
- Cookies are sold singly or in packages of 2 or 6. With this packaging, how many ways can you buy
 - 10 cookies?
 - a dozen cookies?
- In a big red box, there are 7 smaller blue boxes. In each of the blue boxes, there are 7 black boxes. In each of the black boxes, there are 7 yellow boxes. In each of those yellow boxes, there are 7 tiny gold boxes. How many boxes are there altogether? Explain your answer.

- How many triangles are in the following figure?



- Without computing each sum, find which is greater, O or E , and by how much.

$$O = 1 + 3 + 5 + 7 + \dots + 97$$

$$E = 2 + 4 + 6 + 8 + \dots + 98$$

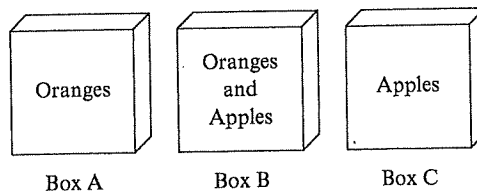
- Alababa, Bubba, Cory, and Dandy are in a horse race. Bubba is the slowest; Cory is faster than Alababa but slower than Dandy. Name the finishing order of the horses.
- How many ways can you make change for a \$50 bill using \$5, \$10, and \$20 bills?
- The following is a magic square (all rows, columns, and diagonals sum to the same number). Find the value of each letter.

17	a	7
12	22	b
c	d	27

- Debbie and Amy began reading a novel on the same day. Debbie reads 9 pages a day and Amy reads 6 pages a day. If Debbie is on page 72, on what page is Amy?
- The 14 digits of a credit card are written in the boxes shown. If the sum of any three consecutive digits is 20, what is the value of A ?

A		7										7		4
-----	--	---	--	--	--	--	--	--	--	--	--	---	--	---

- Three closed boxes (A, B, and C) of fruit arrive as a gift from a friend. Each box is mislabeled. How could you choose only one fruit from one box to decide how the boxes should be labeled?



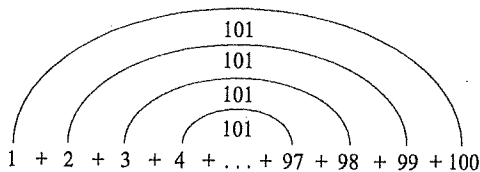
- An electrician charges \$50 per hour and spends \$15 a day on gasoline. If she netted \$1315 in 4 days, how many hours did she work?
- Kathy stood on the middle rung of a ladder. She climbed up three rungs, moved down five rungs, and then climbed up seven rungs. Then she climbed up the remaining six rungs to the top of the ladder. How many rungs are there in the whole ladder?

16. Alex made 4 pies, some apple and some cherry. There were 9 slices in each apple pie and 7 slices in each cherry pie. If he had 34 slices of pie, how many of each type of pie were there?

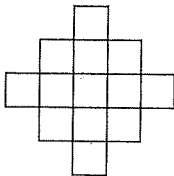
17. Al bought a CD player for \$100, then sold it for \$125. He then bought it back for \$150. Later he sold it for \$175. Did he make money, lose money, or break even? Explain.
18. A baseball bat and ball cost \$50. If the bat costs \$49 more than the ball, what is the cost of each item?

Assessment 1-1B

1. Use the approach in Gauss's Problem to find the following sums of arithmetic sequences.
- $1 + 2 + 3 + 4 + \dots + 49$
 - $1 + 3 + 5 + 7 + \dots + 2009$
 - $6 + 12 + 18 + \dots + 600$
 - $1000 + 995 + 990 + \dots + 5$
2. Use the diagram below to explain how to find the sum of
- the first 100 natural numbers.



- Use this technique to find $1 + 2 + 3 + 4 + \dots + 201$
3. Find the sum of $58 + 59 + 60 + 61 + \dots + 203$.
4. Eve Merriam* titled her children's book *12 Ways to Get to 11* (1993). Using only addition and natural numbers, describe 12 ways that one can arrive at the sum of 11.
5. Explain why in a drawer containing only two different colors of socks one must draw only three socks to find a matching pair.
6. How many squares are in the following figure?



7. If $P = 1 + 3 + 5 + 7 + \dots + 99$ and $Q = 5 + 7 + 9 + \dots + 101$ are sums, determine which is greater, P or Q , and by how much.
8. The sign says that you are leaving Missoula, Butte is 120 mi away, and Bozeman is 200 mi away. There is a rest stop halfway between Butte and Bozeman. How far is the rest stop from Missoula if both Butte and Bozeman are in the same direction?
9. Marc goes to the store with exactly \$1.00 in change. He has at least one of each coin less than a half-dollar coin, but he does not have a half-dollar coin.
- What is the least number of coins he could have?
 - What is the greatest number of coins he could have?
10. Find a 3-by-3 magic square using the numbers 3, 5, 7, 9, 11, 13, 15, 17, and 19.
11. Eight marbles look alike, but one is slightly heavier than the others. Using a balance scale, explain how you can determine the heavier one in exactly three weighings.

12. Recall the song "The Twelve Days of Christmas":
On the first day of Christmas my true love gave to me a partridge in a pear tree.
On the second day of Christmas my true love gave to me two turtle doves and a partridge in a pear tree.
On the third day of Christmas my true love gave to me three French hens, two turtle doves, and a partridge in a pear tree.
 This pattern continues for 9 more days. After 12 days,
- which gifts did my true love give the most? (Yes, you have to remember the song.)
 - how many total gifts did my true love give to me?

13. a. Suppose you have quarters, dimes, and pennies with a total value of \$1.19. How many of each coin can you have without being able to make change for a dollar?
 b. Tell why one of the combinations of coin you have in part (a) is the least number of coins that you can have without being able to make change for a dollar.
14. Suppose you buy lunch for the math club. You have enough money to buy 20 salads or 15 sandwiches. The group wants 12 sandwiches. How many salads can you buy?
15. One winter night the temperature fell 15 degrees between midnight and 5 A.M. By 9 A.M., the temperature had doubled from what it was at 5 A.M. By noon, it had risen another 10 degrees to 32 degrees. What was the temperature at midnight?
16. Seth bought gifts at a toy store and spent \$33. He bought puzzles and trucks. The puzzles cost \$9 each and the trucks cost \$5 each. If he bought 5 gifts, how many of each did he buy?
17. Find the value of the question mark.

+ + = 30

+ + = 18

- = 2

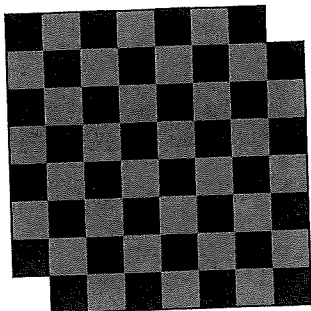
+ = ?

18. You are given a cube that is made of $10 \times 10 \times 10$ smaller cubes for a total of 1000 smaller cubes. If you take off one layer of small cubes all around the larger cube, how many smaller cubes remain?

*Merriam, E. *12 ways to Get to 11*. New York: Aladdin Paperbacks, 1993.

Mathematical Connections 1-1

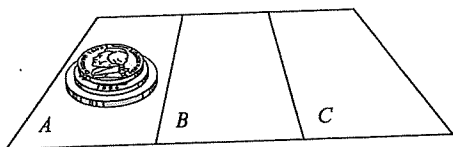
1. Create a 3-by-3 magic square using nine of the ten numbers 20, 21, 22, 23, 24, 25, 26, 27, 28, and 29. Explain your solution and reasoning. List the strategies you have used.



2. In the checkerboard, two squares on opposite corners have been removed. A domino can cover two adjacent squares on the board. Can dominoes be arranged in such a way that all the remaining squares on the board can be covered with no dominoes overlapping or hanging off the board? If not, why not? (Hint: Each domino must cover one black and one red square. Compare this with the number of each color of squares on the board.)
3. a. If 10 people shake hands with one another exactly once, how many handshakes take place?
b. Find as many ways as possible to do the problem.
c. Generalize the solution for n people.
4. An *unmagic square* is one in which the rows, columns, and diagonals must have different sums. Each of the digits 1 through 9 must be used. Complete the unmagic square shown below.

9		7
	1	
3		5

5. Place a half-dollar, a quarter, and a nickel in position A as shown in the figure below. Try to move these coins, one at a time, to position C. At no time may a larger coin be placed on a smaller coin. Coins may be placed in position B.
 - a. How many moves does it take to get them to position C?
 - b. Now add a penny to the pile and see how many moves are required. This is a simple case of the famous Tower of Hanoi problem, in which ancient Brahman priests were required to move a pile of 64 disks of decreasing size, after which the world would end. How long would it take at a rate of one move per second?



6. Choose a problem-solving strategy and make up a problem that would use this strategy. Write the solution using Pólya's four-step approach.
7. The distance around the world is approximately 40,000 km. Approximately how many people of average size would it take to stretch around the world if they were holding hands?

Connecting Mathematics to the Classroom

8. John asks why the last step of Pólya's four-step problem-solving process, *looking back*, is necessary, since he has already given the answer. What could you tell him?
9. A student asks why he just can't make "random guesses" rather than "intelligent guesses" when using the guess-and-check problem-solving strategy. How do you respond?
10. Rob says that it is possible to create a magic square with the numbers 1, 3, 4, 5, 6, 7, 8, 9, and 10. How do you respond?

School Book Pages

11. Refer to the School Book Page on page 5.
 - a. Go through each of the questions asked on the School Book Page and solve the original problem.
 - b. How do the steps laid out on the School Book Page on page 5 compare to Pólya's four-step problem-solving process?
12. Refer to the School Book Page on page 12.
 - a. Solve the *Try It!* problem using only *guess and check*.
 - b. Solve the problem using the given equation.

Group Work

1. Work in pairs on the following versions of a game called NIM. A calculator is needed for each pair.
 - a. Player 1 presses $\boxed{1}$ and $\boxed{+}$ or $\boxed{2}$ and $\boxed{+}$. Player 2 does the same. The players take turns until the target number of 21 is reached. The first player to make the display read 21 is the winner. Determine a strategy for deciding who always wins.
 - b. Try a game of NIM using the numbers 1, 2, 3, and 4, with a target number of 104. The first player to reach 104 wins. What is the winning strategy?
 - c. Try a game of NIM using the numbers 3, 5, and 7, with a target number of 73. The first player to exceed 73 loses. What is the winning strategy?
 - d. Now play Reverse NIM with the keys $\boxed{1}$ and $\boxed{2}$. Instead of $\boxed{+}$, use $\boxed{-}$. Put 21 on the display. Let the target number be 0. Determine a strategy for winning Reverse NIM.
 - e. Try Reverse NIM using the numbers 1, 2, and 3 and starting with 24 on the display. The target number is 0. What is the winning strategy?
 - f. Try Reverse NIM using the numbers 3, 5, and 7 and starting with 73 on the display. The first player to display a negative number loses. What is the winning strategy?
2. Work as a group. You need 15 index cards numbered 1 to 15. Start with a stack of cards numbered in order from least to greatest. Then put the top card (1) face up on the table. Put the next card on the bottom of the stack. Continue to alternate in this way until all cards are face up.
 - a. Suppose you start with 5 cards. Which card do you think will be last? Check your guess.
 - b. Suppose you start with 16 cards. What card will be last?
 - c. How can you predict the last card if you know how many cards you start with?

Solution

One way to solve the problem is to count the matchsticks in rows and columns. The number of columns is the same as the number of rows. Thus, we find the number of matchsticks in the rows and multiply the result by 2. In the first figure, we have 2 rows and 1 matchstick in each. Because we are adding one row and one column to get the subsequent figure, we can write the number of matchsticks in each figure, as shown in Table 16.

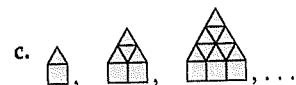
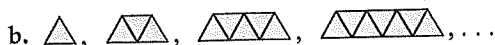
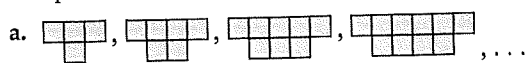
Table 16

Figure Number	Number of Rows	Number in Each Row	Total in the Rows
1	2	1	$1 \cdot 2$
2	3	2	$2 \cdot 3$
3	4	3	$3 \cdot 4$
4	5	4	$4 \cdot 5$
5	6	5	$5 \cdot 6$
\vdots	\vdots	\vdots	\vdots
n	$n + 1$	n	$n(n + 1)$

For part (b), because the number of matchsticks in the columns is the same as in the rows, the total number is $2n(n + 1)$. If $n = 7$, then we have the answer for part (a) as $2 \cdot 7(7 + 1)$ or 112.

Assessment 1-2A

1. For each of the following sequences of figures, determine a possible pattern and draw the next figure according to that pattern:



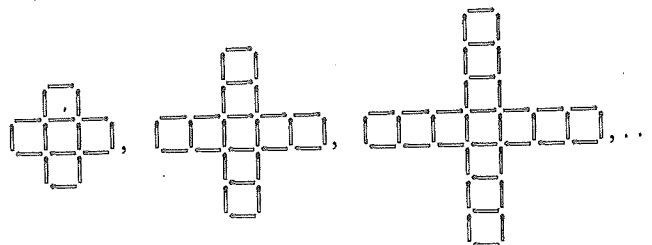
2. Each of the following sequences is either arithmetic or geometric. Identify the sequences and list the next three terms for each.

- 1, 5, 9, 13, ...
- 70, 120, 170, ...
- 1, 3, 9, ...
- $10, 10^3, 10^5, 10^7, \dots$
- $193 + 7 \cdot 2^{30}, 193 + 8 \cdot 2^{30}, 193 + 9 \cdot 2^{30}, \dots$

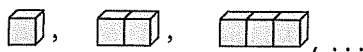
- Find the 100th term and the n th term for each of the sequences in exercise 2.
- Use a traditional clock face to determine the next three terms in the following sequence.

1, 6, 11, 4, 9, ...

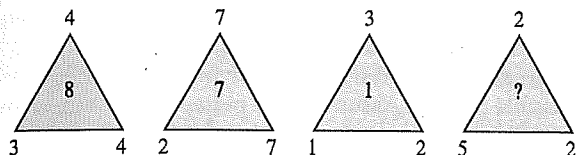
- The pattern 1, 8, 27, 64, 125, ... is a cubic pattern named because $1 = 1 \cdot 1 \cdot 1$ or 1^3 , $8 = 2 \cdot 2 \cdot 2$ or 2^3 , and so on.
 - What is the least 4-digit number greater than 1000 in this pattern?
 - What is the greatest 3-digit number in this pattern?
 - What is the greatest number in this pattern that is less than 10^4 ?
 - If this pattern was produced in a normal spreadsheet, what is the number in cell A14?
- The first windmill has 5 matchstick squares, the second has 9, and the third has 13, as shown. How many matchstick squares are in (a) the 10th windmill? (b) the n th windmill? (c) How many matchsticks will it take to build the n th windmill?



7. In the following sequence, the figures are made of cubes that are glued together. If the exposed surface needs to be painted, how many squares will be painted in (a) the 15th figure? (b) the n th figure?



8. The school population for a certain school is predicted to increase by 60 students per year for the next 12 years. If the current enrollment is 700 students, what will the enrollment be after 12 years?
9. Juan's annual income has been increasing each year by the same dollar amount. The first year his income was \$24,000, and the ninth year his income was \$31,680. In which year was his income \$45,120?
10. Find a number to continue the pattern and replace the question mark. Explain your thinking.



11. One block is needed to make an up-down-up staircase with one step up and one step down.

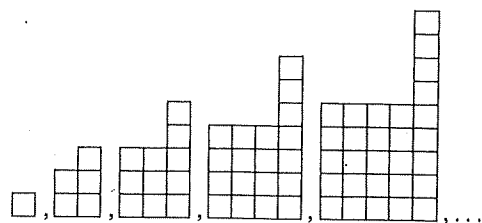


Four blocks are needed to build an up-down-up staircase with 2 steps up and two steps down.



- a. How many blocks are needed to build an up-down-up staircase with 5 steps up and 5 steps down?
- b. How many blocks are needed to build an up-down-up staircase with n steps up and n steps down?
12. How many terms are there in each of the following sequences?
- 51, 52, 53, 54, ..., 251
 - 1, 2, 2^2 , 2^3 , ..., 2^{60}
 - 10, 20, 30, 40, ..., 2000
 - 1, 2, 4, 8, 16, 32, ..., 1024
13. Find the first five terms in sequences with the following n th terms.
- $n^2 + 2$
 - $5n + 1$
 - $10^n - 1$
 - $3n - 2$
14. Find a counterexample for each of the following:
- If n is a natural number, then $(n + 5)/5 = n + 1$.
 - If n is a natural number, then $(n + 4)^2 = n^2 + 4^2$.

15. Assume that the following patterns are built of square tiles and the pattern continues. Answer the questions that follow.



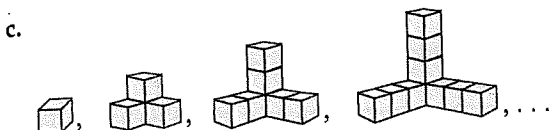
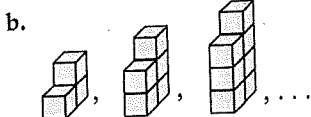
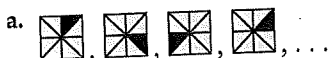
- How many square tiles are there in the sixth figure?
 - How many square tiles are in the n th figure?
 - Is there a figure that has exactly 1259 square tiles? If so, which one?
16. Consider the sequences given in the table below. Find the least number, n , such that the n th term of the geometric sequence is greater than the corresponding term in the arithmetic sequence.

Term Number	1	2	3	4	5	6	...	n
Arithmetic	400	600	800	1000	1200	1400	...	
Geometric	2	4	8	16	32	64	...	

17. A sheet of paper is cut into 5 same-size parts. Each of the parts is then cut into 5 same-size parts and so on. Answer the following.
- After the 5th cut, how many of the smallest pieces of paper are there?
 - After the n th cut, how many of the smallest pieces are there?
18. Each of the following sequences is labeled either arithmetic or geometric. In each part, find the missing terms.
- __, 39, __, __, 69 (arithmetic)
 - __, 200, __, __, 800 (arithmetic)
 - __, 5^4 , __, __, 5^{10} (geometric)
19. A *Fibonacci-type sequence* is a sequence in which the first two terms are arbitrary and in which every term starting from the third is the sum of the two previous terms. Each of the following is a Fibonacci-type sequence. In each part, find the missing terms.
- __, __, 1, 1, __, __, __, __
 - __, __, __, 10, 13, __, 36, 59
 - 0, 2, __, __, __, __, __, __
20. A new pair of tennis shoes cost \$80. If the price increases each year by 5% of the previous year's price, find the following:
- The price after 5 years
 - The price after n years

Assessment 1-2B

1. In each of the following, determine a possible pattern and draw the next figure according to that pattern if the sequence continues.



2. Each of the following sequences is either arithmetic or geometric. Identify the sequences and list the next three terms for each.

a. 2, 6, 10, 14, ...

b. 0, 13, 26, ...

c. 4, 16, 64, ...

d. $2^2, 2^6, 2^{10}, \dots$

e. $100 + 4 \cdot 2^{50}, 100 + 6 \cdot 2^{50}, 100 + 8 \cdot 2^{50}, \dots$

3. Find the 100th term and the n th term for each of the sequences in exercise 2.
4. Use a traditional clock face to determine the next three terms in the following sequence:

$$1, 9, 5, 1, \dots$$

5. Observe the following pattern:

$$1 + 3 = 2^2,$$

$$1 + 3 + 5 = 3^2,$$

$$1 + 3 + 5 + 7 = 4^2$$

- a. Conjecture a generalization based on this pattern.
- b. Based on the generalization in (a), find

$$1 + 3 + 5 + 7 + \dots + 35.$$

6. In the following pattern, one hexagon takes 6 toothpicks to build, two hexagons take 11 toothpicks to build, and so on. How many toothpicks would it take to build
- a. 10 hexagons?
- b. n hexagons?



7. Each successive figure below is made of small triangles like the first one in the sequence. Conjecture the number of small triangles needed to make
- a. the 100th figure?
- b. the n th figure?



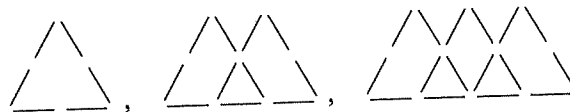
8. A tank contains 15,360 L of water. At the end of each subsequent day, half of the water is removed and not

replaced. How much water is left in the tank after 10 days?

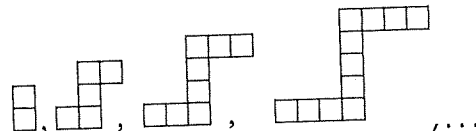
9. The Washington Middle School time schedule is an arithmetic sequence. Each period is the same length and includes a 4th period lunch. The first three periods begin at 8:10 A.M., 9:00 A.M., and 9:50 A.M., respectively. At what time does the eighth period begin?
10. There are nine points drawn as shown below. Can you connect all nine points with four straight line segments without lifting a pen from the paper.



11. The triangular figures shown below are constructed with toothpicks. The pattern shows what happens with 1 triangle, with 2 triangles, and with 3 triangles. Describe a possible pattern. Assuming the pattern continues, answer the following questions.



- a. How many toothpicks would be needed for the 10th figure?
- b. How many toothpicks are needed for the n th figure?
- c. In which figure are exactly 102 toothpicks used?
12. How many terms are there in each of the following sequences?
- a. $1, 3, 3^2, 3^3, \dots, 3^{99}$
- b. $9, 13, 17, 21, 25, \dots, 353$
- c. $38, 39, 40, 41, \dots, 238$
13. Find the first five terms in sequences with the following n th terms.
- a. $5n - 1$
- b. $6n - 2$
- c. $5n + 1$
- d. $n^2 - 1$
14. Find a counterexample for each of the following:
- a. If n is a natural number, then $(3 + n)/3 = n$.
- b. If n is a natural number, then $(n - 2)^2 = n^2 - 2^2$.
15. Assume the following pattern with terms built of square tiles continues and answer the questions that follow.



- a. How many square tiles are there in the seventh figure?
- b. How many square tiles are in the n th figure?
- c. Is there a figure that has exactly 449 square tiles? If so, which one?
16. Consider the sequences given in the table below. Find the least number, n , such that the n th term of the geometric

sequence is greater than the corresponding term in the arithmetic sequence.

Term Number	1	2	3	4	5	6	...	n
Arithmetic	200	500	800	1100	1400	1700	...	
Geometric	1	3	9	27	81	243	...	

17. Female bees are born from fertilized eggs, and male bees are born from unfertilized eggs. This means that a male bee has only a mother, whereas a female bee has a mother and a father. If the ancestry of a male bee is traced 10 generations including the generation of the male bee, how many bees are there in all 10 generations? (Hint: The

Fibonacci sequence might be helpful.)

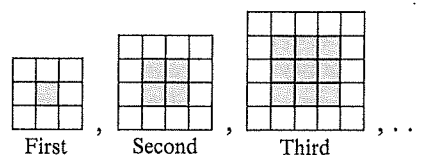
18. Each of the following sequences is labeled either arithmetic or geometric. In each part, find the missing terms.
- _, 49, _, 64 (arithmetic)
 - 1, _, 625 (geometric)
 - _, 3^{10} , _, 3^{19} (geometric)
 - a , _, 5a (arithmetic)
19. Each of the following sequences is a Fibonacci-type sequence. Find the missing terms.
- 1, _, 7, 11
 - _, 2, 4, _
 - _, 3, 4, _

Mathematical Connections 1-2

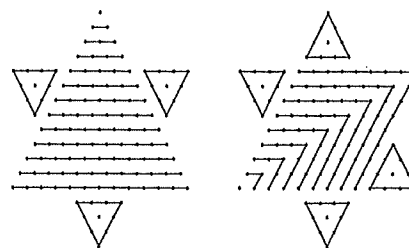
- If a fixed number is added to each term of an arithmetic sequence, is the resulting sequence an arithmetic sequence? Justify the answer.
 - If each term of an arithmetic sequence is multiplied by a fixed number, will the resulting sequence always be an arithmetic sequence? Justify the answer.
 - If the corresponding terms of two arithmetic sequences are added, is the resulting sequence arithmetic?
- A student says she read that Thomas Robert Malthus (1766–1834), a renowned British economist and demographer, claimed that the increase of population will take place, if unchecked, in a geometric sequence, whereas the supply of food will increase in only an arithmetic sequence. This theory implies that population increases faster than food production. The student is wondering why. How do you respond?
- Abby and Dan are preparing for a GRE (Graduate Record Exam) to take place in 5 months. Abby starts by studying 10 hours the first week and increases her study by 30 minutes per week. Dan starts at 6 hours per week, but increases his time every week by 45 minutes per week. Answer the following.
 - How many hours did each student study in week 8?
 - In which week will Dan first catch up with Abby in the number of hours spent studying per week?
- The arithmetic average of two numbers x and y is $\frac{x+y}{2}$. Use deductive reasoning to explain why if three numbers a , b , and c form an arithmetic sequence, then b is the arithmetic average of a and c .
- A mathematician named Christian Goldbach (1690–1764) made a conjecture that has not been proven. Millions of examples have been found that support his conjecture, but no counterexample has ever been found. Until a counterexample is found or someone presents a logical proof, this will remain a conjecture.
Goldbach's Conjecture: Every even number greater than 2 can be written as a sum of two prime numbers.

Test Goldbach's Conjecture for various even numbers, for example, $4 = 2 + 2$ and $10 = 7 + 3$.

6. The figure below shows the first three terms of a sequence of figures containing small square tiles. Some of the tiles are shaded. Notice that the first figure has one shaded tile. The second figure has $2 \cdot 2$, or 2^2 , shaded tiles. The third figure has $3 \cdot 3$, or 3^2 , shaded tiles. If this pattern of having shaded squares surrounded by white borders continues, answer the following:
- How many shaded tiles are there in the n th figure?
 - How many white tiles are there in the n th figure?



7. Patterns can be used to count the number of dots on the Chinese checkerboard; two patterns are shown here. Determine several other patterns to count the dots.



$$1 + 2 + 3 + \dots + 13 + 3 \cdot 10 \quad 1 + 3 + 5 + 7 + \dots + 17 + 4 \cdot 10$$

8. Make up a pattern involving figurate numbers and find the number of dots in the 100th figure. Describe the pattern and how to find the 100th term.

Connecting Mathematics to the Classroom

9. Joey said that 4, 24, 44, and 64 all have remainder 0 when divided by 4, so all numbers that end in 4 must have 0 remainder when divided by 4. How do you respond?